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# SCHOOL SCIENCE AND MATHEMATICS

OCTOBER 1960

# School Science and Mathematics

***A Journal for All Science and Mathematics Teachers***

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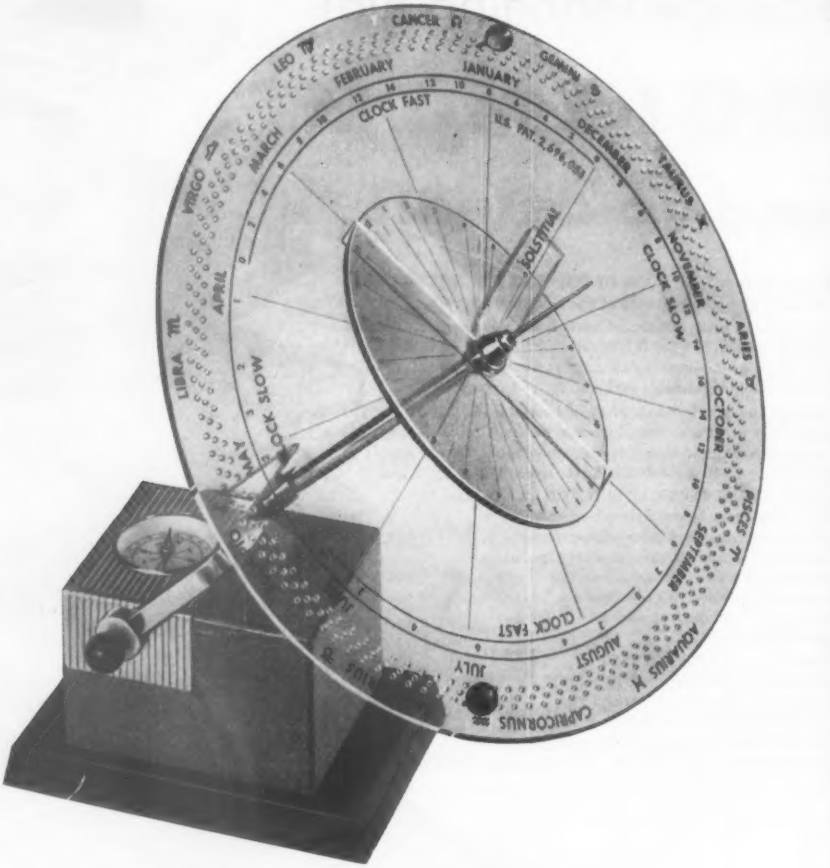


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# SCHOOL SCIENCE AND MATHEMATICS

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## Caveat Emptor: An Old Problem Made New by NDEA

Ralph P. Frazier

*Specialist, Science Equipment and Materials; Science, Mathematics, and Foreign Language Section, Dept. of Health, Education and Welfare, Office of Education, Washington 25, D. C.*

A transfusion of dollars has suddenly been introduced into the scientific equipment field to overcome a dangerous case of anemia. Millions of dollars are being spent to up-date, refurbish and equip laboratories and classrooms with the latest science equipment and facilities. A substantial proportion of this money is being supplied by Title III of the National Defense Education Act.

During the fiscal year ending June 30, 1959, the Federal Government allocated $49,280,000 to the States for the purpose of making grants to public elementary and secondary school units to purchase science, mathematics and modern foreign language equipment. An even greater sum, $52,800,000, is being provided this year. This is but half the story. For every one of the Federal dollars expended an additional dollar is also expended from State and local sources. This means that there were available in fiscal year 1959, $98,560,000 and in 1960, $105,600,000. And expenditures of this magnitude will probably continue for at least two more years. Parenthetically, it should be noted that a State's allocation in one fiscal year for equipment acquisitions can be carried over and spent during the following fiscal year.

On the basis of reports submitted to the U. S. Commissioner of Education by the State educational agencies 76.6% of the money spent during the fiscal year 1959 went into science equipment and minor remodeling necessary to make effective use of the equipment

acquired. There are indications from the same reports that the proportion of money which will be spent for science equipment in succeeding years will not be as great.

It is very difficult to judge the amounts of money expended on science equipment and facilities before the advent of the NDEA. According to an estimate of the Research Division of the National Education Association (1) approximately $30,000,000 was spent in the fiscal year prior to the spring of 1958 for science equipment in the public secondary schools. No estimates are available as to the amount of money spent by the public elementary schools for comparable items. No doubt such expenditures are included in elementary school budgets without definition or categorization. Traditionally science in the elementary school has never had the same financial status or support possessed by secondary school science. On this basis it is probably safe to assume that a smaller amount of money has been spent each year on elementary school science programs than on secondary school science programs. If this assumption is correct the combined expenditures for both elementary and secondary science in the public schools has been considerably less than the amount now available from Federal, State and local sources for the same purposes.

As more funds become available there is increased concern with the manner in which these funds are to be spent. The need for criteria and standards by which to make judgments both as to what should be purchased and as to the specifications of individual items has gained increasing recognition. In a recent article, George R. Larke (2) stressed the need for obtaining the "maximum ultimate value for each dollar of expenditure." He reiterates the thesis that it is absolutely necessary to establish specifications for products to be purchased and to conduct performance tests to be certain that the specifications have been met.

Two documents have been published which will be of value to teachers and administrators concerned with the purchasing of science equipment. The first is "Action for Science Under NDEA" published by the NSTA (3). This pamphlet points out that "purchases should be consistent with the program offered and reflect genuine instructional needs." It goes on to provide a set of principles for the guidance of those persons considering the purchase of equipment and materials.

Possibly the most valuable recent publication is the "Purchase Guide for Programs in Science, Mathematics and Modern Foreign Languages," prepared by the Council of Chief State School Officers (4) with the assistance of many other individuals and agencies including the Educational Facilities Laboratories, Inc. Therein are included not only suggested lists of science equipment, but more importantly, the suggested numbers, uses and specifications for each item listed.

This is a very useful publication and should be consulted fully before selection of items for purchase are made. Note should be taken that some items included in the Guide are not eligible for purchase with NDEA funds.

Just as every coin has two faces, so does NDEA. On one side is seen the face of opportunity: opportunity for the improvement of science instruction through the purchase of needed science equipment. The opposite face of the coin presents responsibility: responsibility which must be exercised at several levels. Teachers, supervisors, principals, superintendents and State educational personnel, must all share in the responsibility of using the available funds judiciously and wisely. To dissipate the money in a profligate manner on non-essential or unwise choices of equipment or materials is to fail in this inherent responsibility. Although more money is now available for these purposes than ever before, careful evaluation of needs and of the expenditures to meet these needs is essential. Sound stewardship is necessary both to maintain the public confidence and to derive the absolute maximum in educational facilities and opportunities from the funds available.

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## RUSSIANS FIND MAGNETISM INFLUENCES PLANTS, ANIMALS

All living things—including man—have north and south poles of sorts, two Russian scientists believe.

They have discovered that the earth's magnetic field exerts a definite effect on growth processes in plants. Establishing the existence of this phenomenon—magnetotropism—makes the Russians sure that polarity is a fundamental property of all living material.

They report magnetotropism may influence cancer and radiation effects.

Magnetotropism is defined as consisting of the oriented growth in the direction of the earth's south magnetic pole or an artificial south magnetic pole. It is similar in effect to phototropism, the attraction that makes a houseplant grow toward a sunny window.

Research showed that magnetic influence on plant root growth is greatest when seedlings are in the embryo stages, and when these embryos are oriented along an imaginary line drawn from pole to pole, rather than crosswise of the magnetic field.

When a seedling has its roots directed toward a natural or artificial south pole, both the roots and stem swell up and grow faster than usual. When turned with roots toward a north pole, the seedlings grow slower.

# Joseph Priestley

**Fred Meppelink**
*Science Teacher West Ottawa School, Holland, Michigan*

Joseph Priestley[1] was born into a family of cloth makers on March 13, 1733. His father died at an early age and Joseph spent most of his early years with his maternal grandfather, a farmer named Swift. Later he was adopted by an aunt named Keighly, the wife of a wealthy merchant. His aunt provided for his education by sending him to several schools where he learned Greek, Latin and by private tutoring Hebrew. All this of course in preparation for the ministry.

His family was of the Calvinistic faith and of course young Priestley was instructed in their doctrines. All of his early life he planned on taking up the ministry, but at the age of sixteen he showed signs of consumption and had to give up his plans. He set about to learn French, Italian and High Dutch, without the benefit of a master. This he did and is certainly an indication of the caliber of this remarkable young man. He used these language skills to translate letters for his merchant uncle, who planned on sending him to Portugal as his representative.

Suddenly his health improved and he could again continue his preparation for the ministry, in a school at Daventry. At this school he had training in Geometry, Algebra and other mathematics, as well as Natural Philosophy and the languages Chaldee, Syriac and Arabic.

Preliminary schooling finished, it came time to enter what would now be called a seminary. This posed somewhat a problem for Priestley because although born into Calvinism, he had long since abandoned some of the doctrines of that faith. He refused to even attempt entry into the Calvinistic school because he knew he would be examined and found lacking in zeal for the doctrines. He had dropped from his credo the Doctrine of Original Sin and had grave doubts about several others. He finally entered a dissenters academy at Northampton and was subsequently graduated as a novice minister.

Priestley's first congregation was at Needham Market. This was a very small and poor church which received partial support from the Presbyterian and Independent Fund. Priestley immediately refused these moneys because he did not ascribe to their beliefs. This fact, coupled with the fact that his heresy caused the people of the church

[1] Joseph Priestley, a pneumatic chemist of the 18th Century, was responsible for the isolation and characterization of more gases than any of his contemporaries. These many experiments by Priestley enabled Lavoisier to lay the foundation of modern chemistry. This account of Priestley's life and work was prepared by the author in the Spring of 1959 while a member of the "History of Chemical Theory" class at Western Michigan University.

to refuse to support the church to any great extent, forced Priestley to take on the additional job of schoolmaster. He did not like school teaching but he had remarkably good success in spite of this, and apparently even overcame the obstacle of a severe speech defect, which handicapped him all his life. His teaching methods seemed to have been closely akin to what is now called progressive education.

Finally he left Needham Market and in 1758, deeply disappointed in his first experience as a minister, accepted the pastorate at Nantwich, an even smaller congregation. However the people there were more congenial to his religious views and in spite of the fact that he had to teach to keep from starving, as well as tutor up to twelve hours a day, he enjoyed his three years there.

His successful teaching procedures led to his appointment as tutor to Warrington Academy in 1761. It was in this period, from 1761 to 1767, that Priestley wrote many books and dissertations on history, government and human rights. From Warrington he yearly took off one month for study in London and it was during these visits that he became acquainted with Benjamin Franklin. It was Franklin who introduced him to scientific theory and finally persuaded him to write a history of electricity. In spite of the fact that his academic duties took up ten hours of his day, Priestley completed the history in a year.

In 1765, Priestley had been made LLD by the University of Edinburgh, in recognition of his Chart of Biography. With the publication of his history of electricity he attracted enough attention in the science world to be appointed to the Royal Society in 1766. In spite of all these honors his salary at Warrington Academy was not large enough to support his family. In 1767, he decided to have his third try at the ministry and went to Leeds to accept the pastorate at Mill Hill.

In the period of his life from 1767 to 1773 Priestley devoted much of his time to writing theological works, but still had time for some scientific experimentation. As the story goes, his first home in Leeds was next to a brewery and Priestley would amuse himself by experimenting with the "fixed air"[2] which he found was formed in the fermentation process used in beer making. These were among his first experiments with the gases. He later moved away from the brewery and was forced to generate "fixed air" himself in order to carry out his tests.

Priestley's first publication in the Chemistry field was entitled "Directions for Impregnating Water with Fixed Air." This was published in 1772 and attracted widespread attention. Some people in the British navy became convinced that water so treated might be of value in preventing scurvy in ships' crews and some vessels were

[2] Fixed air = carbon dioxide.

equipped with apparatus following Priestley's design.

The notoriety following his "fixed air" publication led to his appointment as Lord Shelburne's personal librarian and literary companion. It was during this period of his life, from 1772 to 1780, that the great preponderance of Priestley's scientific experimentation was carried out. Previous to 1772 he had perfected a method of collecting gases over mercury instead of water, and it was this technique which lead him to isolate hydrogen chloride and ammonia gas. By heating common salt with vitriolic acid he prepared an acid which he called "marine acid" which today is known commercially as muriatic acid and has wide use in industry.

Priestley's greatest contribution however, came on August 1, 1774 when he effectively discovered oxygen. He of course was not aware of the nature of his discovery, and being a phlogistonist, called his gas "dephlogisticated air." His reasoning for calling it by this name followed these lines; because this new air caused a candle's flame to burn much brighter, it must contain no phlogiston and for this reason the candle could freely give up it's phlogiston and thus burn with greater vigor. He regarded the new gas as merely a better air, which he substantiated by experimentation with mice. The animals, of course lived longer in the gas than they did in common air, therefore he concluded that the air he had generated from the mercurius calcinatus was of a higher quality, Priestley, in his report of the experiment, attributes his application of the candle to the gas as a mere chance happening and that the candle just happened to be in front of him and that he used no concious effort to use it in a test. This may very well be true because throughout his writings he unabashedly reports his foolish mistakes as well as his successes.

Priestley's arch rival now enters the scene, Lavoisier. In his capacity as Lord Shelburne's literary companion, Priestley had many opportunities to travel on the Continent. On one of these sojourns, shortly after his experiments with mercuric oxide, he met Lavoisier the eminent French chemist, and being a trusting soul, described his procedure for producing "kephlogisticated air." He explained that he believed some of the difficulties he had with quantitative measurements in the experiment, resulted from the fact that his sample of mercurius calcinatus was not pure. He also explained that he was of the opinion that a French apothecary named Cadet produced a very high grade of this chemical and that while in Paris he intended to acquire some of his product.

Lavoisier apparently, immediately set about to repeat Priestley's experiment and of course duplicated his results. This of course in the finest scientific tradition. However, Lavoisier apparently pulled off a

bit of plagiarism because later in his book *Elements of Chemistry* he states that he and Priestley accomplished this experiment at the same time. This Priestley later pointed out in another of his publications and for some time a hot argument ensued. This argument eventually developed into a running debate on the relative merit of the phlogiston theory. Priestley of course affirmative and the Frenchman, Lavoisier the negative. Lavoisier, of course, eventually discredited it but in spite of obvious evidence Priestley stubbornly refused to accept facts and held to it until his dying day.

Priestley's important chemical publications were contained in his six volumes on air. The First Volume recounts his experiments with "burned out air" and "air infected with animal respiration," as well as other early work. The second volume, and most important, reported his experiments leading to the discovery of oxygen. The third volume include "Experiments on the Mixture of Different Kinds of Air" in which he indicates the principle of intradiffusion of gases. In volume four he relates experiments with the antiseptic action of nitric oxide. He attempted the preservation of meat by holding it for some time in the gas. The fifth volume contains the discovery of nitrous oxide. The sixth volume was the least important and dealt primarily with unsuccessful attempts to convert water into air.

Lord Shelburne retired Priestley in 1780 with a pension, and he returned to his first love, the ministry. He accepted a church at Birmingham and in spite of his heretical preaching attracted quite a large following. These were stormy times in England and in 1891 a mob burned down his chapel and he was forced to flee for his life. He was an outspoken supporter of both the French Revolution and the United States, and therefore was not looked on with too much favor by the English royal family. However he later sued for damages inflicted by the mob and received a cash settlement from the city of Birmingham. Three years after the burning however he decided to emmigrate America.

Priestley lived for a short time in New York and Philadelphia but because he was disappointed by not being offered a pulpit, he eventually settled in a wilderness town called Northumberland, Pennsylvania. He contributed practically nothing to science in his remaining life in this new land and because of the remoteness of his settling place was unable even to keep up with progress. He remained a staunch phlogistonist to his death which came in Northumberland February 6, 1804.

In reflecting, one wonders what accomplishments this amazing man could have achieved had he spent all his life at scientific endeavor, for in reality his scientific career covered only a brief seven years.

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## JOHN HAY FELLOWSHIPS FOR 1961–62

Seventy-five John Hay Fellowships for 1961–62 will be awarded to public senior high school teachers by the John Hay Fellows Program. Winners of these awards will study in the humanities for a year at one of the following Universities: California, Chicago, Columbia, Harvard, Northwestern, and Yale. They will receive stipends equal to their salaries during the fellowship year. In addition, travel expenses, tuition, and a health fee will be paid.

The seventy-five John Hay Fellows will be selected from schools and school systems interested in making the best possible use of the time and talents of good teachers and in developing practices designed to break educational lock steps. Applicants should have at least five years of high school teaching experience, and should be not more than fifty years of age.

Five new states will participate in the John Hay Fellows Program for 1961–62: California, Florida, Indiana, New Hampshire, and Wisconsin. The other participating states are: Arizona, Colorado, Connecticut, Illinois, Louisiana, Maryland, Massachusetts, Michigan, Missouri, New York, Ohio, Oregon, Pennsylvania, Utah, and Virginia, as well as the District of Columbia.

Languages, literature, history, music, and the fine arts are usually included in the humanities, and teachers of these subjects are invited to apply. In addition, applications from teachers in other disciplines who wish to study in the humanities are accepted.

The John Hay Fellows Program received a new grant from the Ford Foundation last spring which will enable it to continue its activities through 1966. The Program was established in 1952 by the John Hay Whitney Foundation.

Interested teachers should communicate with Dr. Charles R. Keller, Director, John Hay Fellows Program, 9 Rockefeller Plaza, New York 20, New York. Applications will close on December 1, 1960.

## GIBBERELLIN FEED PATENTED, MAY SPEED ANIMAL GROWTH

Gibberellin, a non-antibiotic material that increases the growth rate of plants, may be added to animal feed to give better and faster yields.

Scientists state that the addition of very small amounts of gibberellin, between one part in a billion and one in a million, is effective in increasing the growth rate of poultry and is also effective for cattle, sheep, pigs, lambs and other farm animals.

The use of various hormones, and of antibiotics, in animal feeds is not new but, according to the inventors, gibberellin, a term they use to include the three compounds known specifically as gibberellins A-1, A-2, and A-3, is much more effective than such previously used materials as penicillin and stilbestrol, in promoting growth. Since it is not an antibiotic, gibberellin does not reduce the incidence of disease, as does penicillin for example, so these other additives may still be necessary in some cases.

# A Teaching Unit in Modular Arithmetic for Grade Eight

Margaret F. Willerding
*San Diego State College, San Diego, California*

## I. Introduction

The eighth grade mathematics program, along with developing better skills in working with numbers, should be an introductory or experimental period for the student. The student should be given an opportunity to investigate other mathematical ideas. If the student continues on a mathematic program through high school and even college, these ideas will be brought up again and again. The terms and concepts will become more familiar to him each time he comes into contact with these ideas. For the student who will perhaps receieve little or no more math training, these ideas will at least give him insight into some of the problems in mathematics. The unit outlined here then, is to present the modular number system in an introductory manner.

## II. Objectives

A. To introduce the student to a new number system and to show him that there are other mathematical systems from what he has known.

B. To introduce the student to new terms which he will see again and again in mathematics.

C. To increase their capacity with working with simple algebraic concepts.

D. To use some examples in the student's environment so he can perhaps use these ideas himself at a later time.

E. To increase each student's skill in working with numbers in general.

## III. Procedures

A. Introduce the unit with the calendar dial. (See Fig. 1.)

1. Explain the dial and explain that each day can be called a number (instead of Sunday, call it 0, etc.).
2. Ask the students what day we get when we start at a certain day and go a certain number ahead. Do this with several different days and numbers to see that everyone understands the technique. Show that this is a kind of addition.
3. Now find out what happens when we go back a certain number from a certain day. Show that this is a kind of subtraction.
4. Next use the calendar dial for showing repeated addition, that is go to a certain number then twice that number, three times that number, etc. Show that this is a kind of multiplication.

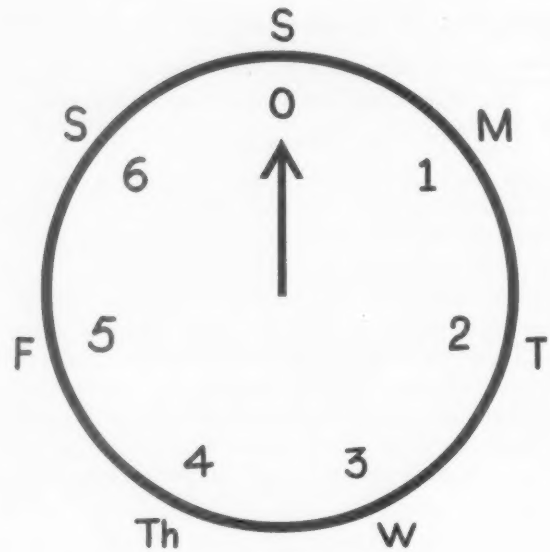


FIG. 1

5. Bring in the name modular to familiarize them with the term right away. We have been working with modular seven because there are seven numbers that we use.
6. Ask for suggestions for other modular systems (perhaps have some other dials handy for use).
   (Note: At this grade level perhaps subtraction and division could be just introduced and only some of the better students emphasized to work with these operations.)

B. Building an addition and multiplication table.
   1. Use the whole class, and the calendar dial to fill the tables. The students will begin before finished, to see the results, but encourage them to use the dial in each case to make sure.
   2. Make sure each student has a copy of the table for future reference.

C. Use of the table.
   1. Ask questions of the class, what we get when we add two numbers; multiply two numbers. Continue about the class to ascertain whether the table is read correctly or not.
   2. Bring up the question of how we can add three numbers; multiply three numbers. Do we add all three at once? Do we add the first two, then the third, or do we add the last two and add that to the first? If the associative law has been

brought out before this unit, show that it holds for modular numbers.

3. When we add or multiply two numbers does it make a difference in what order they are taken? Show the commutative law hold.
4. Discuss how we can subtract or divide using the table.

D. Using letters instead of numbers.
1. Show that we get a number in our system by use of the tables whenever we add, subtract, multiply or divide. Show that the closure law holds.
2. Discuss the associative law with letters instead of numbers.
3. Discuss the commutative law with letters instead of numbers.
4. Show that when we add zero to any number we always get that number. Show that when we multiply by one any number, we always get that number. Show that there is an identity element for each operation.
5. Show that there is some number for each number that when added together gives us zero. Show that there is some number for each number that when multiplied together gives us one. Show that this is the inverse element for each operation.

E. Distributive law.
1. Show that $a(b+c)=ab+bc$. Have the students try this for many numbers in the system.

F. Some further ideas for better students.
1. Does the reverse distributive law work? (i.e. does $a+(bc)=(a+b)\ (a+c)$?
2. What happens, using the multiplication table when we divide a number by zero? Discuss the prime numbers.
3. If the students have had some work with simple linear equations, consider some equations with modular numbers.
4. Perhaps show what happens when we take powers of the numbers in our new system and continue until we have repeating powers.

## IV. Assignments for the Students

A. Assign students to make a dial for numbers other than modulo 7 (preferably less than 10). Add and multiply in their system each of the numbers to familarize them with the concepts of the modular number system.

B. Make a table of addition and multiplication using the numbers modulo 3 and modulo 4.

C. 1. Give the students an exercise of problems dealing with the

modular 3 system and modular 4 system using the table they have built.

(a) $1+2=?$ (mod 4)
(b) $2+1=?$ (mod 3)
(c) $3\times3=?$ (mod 4)
(d) $0+3=?$ (mod 4)
(e) $0+(3+2)=?$ (mod 4)
(f) $1\times(2\times0)=?$ (mod 3)
(g) $(1+1)+1=?$ (mod 3)
(h) $(2+3)+0=?$ (mod 4)
(i) $1+(1+1)=?$ (mod 3)
(j) $0+(2+0)=?$ (mod 3)

2. True or False
(a) $1+2=2+1$ (mod 3)
(b) $1\times2=2\times1$ (mod 4)
(c) $3+(2+1)=(2+2)+1$ (mod 4)
(d) $(1+2)+2=3+(2+0)$ (mod 4)
(e) $(0\times3)=(2\times2)$ (mod 4)
(f) $(2+1)+1=2+(2+0)$ (mod 3)
(g) $(6+5)=(4+0)$ (mod 7)
(h) $(6\times4)=(3\times2)$ (mod 7)
(i) $(2\times1)=(3\times3)$ (mod 7)

3. Extra
(a) $3-2=?$ (mod 4)
(b) $3\div2=?$ (mod 7)
(c) $4-5=?$ (mod 7)
(d) $2\div1$ (mod 3)
(e) $4+(1-2)=?$ (mod 7)
(f) $2\div2=?$ (mod 4)

D. 1. In each case let $a=1$, $b=3$, and $c=0$.
(a) $a+b=?$ (mod 4)
(b) $c+a=?$ (mod 3)
(c) $ab=?$ (mod 7)
(d) $a(bc)=?$ (mod 7)
(e) $(ab)c=?$ (mod 7)
(f) $a+(a+b)=?$ (mod 4)
(g) $a+(c+b)=?$ (mod 4)
(h) $b+(c+0)=?$ (mod 7)

2. Extra
(a) $a-c=?$ (mod 3)
(b) $c-a=?$ (mod 4)
(c) $a-b=?$ (mod 7)
(d) $a\div b=?$ (mod 4)
(e) $(a-b)+c=?$ (mod 4)
(f) $c(b\div a)$ (mod 7)

3. Using $a=3$, $b=4$, $c=6$ modulo 7, show that
(a) $(a+b)+c=a+(b+c)$
(b) $(ab)+c=(a\times b)+c$
(c) $ac=ca$
(d) $(bb)b=b(bb)$
(e) $(a+c)+a=(a+a)+c$
(f) $a(bb)=(ab)b$
(g) $a+(b+a)=a+(a+b)$
(h) $(cc)a=c(ca)$

4. Using $a=0$, then $a=1$, then $a=2$ (modulo 3) show that
(a) $a+0=0+a=a$
(b) $a\times1=1\times a=a$

5. Find the inverse element for each case.
(a) $3+$____$=4$ (mod 7)
(b) $2\times$____$=3$ (mod 4)
(c) $0+$____$=0$ (mod 3)
(d) $1\times$____$=1$ (mod 4)

E. 1. Find the answer.
(a) $3\times(2+3)=$____ (mod 4)
(b) $2\times(2+2)=$____ (mod 7)
(c) $1\times(2+3)=$____ (mod 7)
(d) $0\times(6+5)=$____ (mod 7)

(e) $1\times(0+2)=$____ (mod 3) (g) $5+(2\times1)=$____ (mod 7)
(f) $6\times(2+3)=$____ (mod 7) (h) $2+(3\times2)=$____ (mod 4)

2. Let $a=0$, $b=2$, $c=1$ (modulo 3) show that:
   (a) $a(b+c)=ab+ac$ (e) $c(a+b)=cb+ca$
   (b) $a(c+b)=ac+ab$ (f) $c(a+a)=ac+ac=2ac$
   (c) $a(c+c)=ac+ac=2ac$ (g) $c(a+0)=ca+0=0$
   (d) $b(a+c)=ba+bc$ (h) $b(b+b)=bb+bb=2b^2$

F. Some advanced problems
1. True or false.
   (a) $3\times(2+3)=3\times2+3\times3$ (mod 4)
   (b) $3\times(2+5)=3\times2+3\times5$ (mod 7)
   (c) $4\times(2+2)=4\times2+4\times3$ (mod 7)
   (d) $2\times(2+1)=3$ (mod 4)
   (e) $2+(3\times2)=(2+3)\times(2+2)$ (mod 4)
   (f) $5+(6\times1)=(5+6)\times(5+1)$
2. Give the answer if there is *one*.
   (a) $0\div2=$____ (mod 3) (e) $1\div2=$____ (mod 4)
   (b) $6\div0=$____ (mod 4) (f) $1\div2=$____ (mod 3)
   (c) $5\div4=$____ (mod 7) (g) $1\div0=$____ (mod 3)
   (d) $5-6=$____ (mod 7) (h) $0-4=$____ (mod 7)
3. Find $x$.
   (a) $x+\mathrm{a}=3$ (mod 4) (e) $5x=6$ (mod 7)
   (b) $x-1=3$ (mod 4) (f) $2x+1=1$ (mod 3)
   (c) $3-x=3$ (mod 7) (g) $x-2=2$ (mod 4)
   (d) $2x=3$ (mod 3) (h) $2x=1$ (mod 4)
4. Find the powers of the numbers listed below.
   (a) $0^2=$____ (mod 3) (f) $3^2=$____ (mod 4)
   (b) $1^2=$____ (mod 3) (g) $2^3=$____ (mod 7)
   (c) $2^2=$____ (mod 3) (h) $2^4=$____ (mod 7)
   (d) $2^3=$____ (mod 3) (i) $2^5=$____ (mod 7)
   (e) $2^4=$____ (mod 3) (j) $2^6=$____ (mod 7)

## V. Conclusion

A test will be given at the end of the unit to evaluate the concepts learned. After the test is given each student will have the opportunity to go through and make corrections for the next time. Those topics that need further consideration and understanding will then be discussed. The test will include the addition and multiplication tables for modular 7 and modular 4.

*Math 8*

*Modular Numbers* *Unit Test*

A. True—False: If true mark in A on the answer sheet, if false mark in B.

1. $3+2=2+3 \pmod 4$ (true)
2. $1+(2+4)=(1+2)+4 \pmod 7$ (true)
3. $2+(3+4)=1 \pmod 7$ (false)
4. $1\times 2=2 \pmod 4$ (true)
5. $3\times 3=0 \pmod 4$ (false)
6. $3\times 3=0 \pmod 7$ (false)
7. $2\times(3+4)=0 \pmod 7$ (true)
8. $2\times(2+2)=2\times 2+2\times 2 \pmod 4$ (true)
9. $2\times(5+6)=2\times 4 \pmod 7$ (true)
10. $2+(3\times 3)=(2+3)\times(2+3) \pmod 4$ (false)
11. $0+2=3+1 \pmod 7$ (true)
12. $0+2=3+1 \pmod 7$ (false)
13. $2+(1+1)=2+3) \pmod 4$ (false)
14. $a+b=b+a \pmod 4$ (true)
15. $a+(b+c)=(a+b)+c \pmod 7$ (true)
16. $ab=ba \pmod 7$ (true)
17. $a(bc)=(ab)c \pmod 4$ (true)
18. $a(b+c)=ab+bc \pmod 7$ (false)
19. $2^2=4 \pmod 4$ (false)
20. $3^2=3^4 \pmod 4$ (true)

B. Multiple choice. Choose the best answer.

1. $3+4=$____ (mod 7)
   (a) 7 (b) 5 (c) 1 (d) *0*
2. $3+2=$____ (mod 4)
   (a) *0+1* (b) 0+2 (c) 2+2 (d) none of these.
3. $3+4$____ (mod 7)
   (a) 0+1 (b) 2+4 (c) *4+3* (d) none of these.
4. $2+3=$____ (mod 5)
   (a) 1 (b) *0* (c) 5 (d) 6
5. $2+(3+4)=$____ (mod 7)
   (a) 0 (b) 5 (c) 9 (d) *none of these.*
6. $5\times 4=$____ (mod 7)
   (a) 2 (b) 0 (c) *6* (d) 1
7. $2\times 3=$____ (mod 5)
   (a) *1* (b) 0 (c) 6 (d) 2
8. $4(3+2)=$____ (mod 7)
   (a) 2 (b) *6* (c) 20 (d) none of these.
9. $4\div 0=$____ (mod 7)
   (a) 4 (b) 3 (c) 0 (d) *none of these.*
10. $5\times(4+3)=$____ (mod 7)
    (a) *0* (b) 35 (c) 5 (d) none of these.
11. $2^3=$____ (mod 4)
    (a) 8 (b) 1 (c) 2 (d) *0*

12. $4 \div 2 =$ ____ (mod 4)
    (a) 1 (b) 3 (c) 5 (d) ***none of these.***
13. $2^3 =$ ____ (mod 3)
    (a) 0 (b) 1 (c) *2* (d) none of these.
14. $a+b=$ ____
    (a) $ab$ (b) $b+a$ (c) $2a$ (d) none of these.
15. $a(b+c)=$ ____
    (a) $a+b+c$ (b) $(a+b)+c$ (c) $a(bc)$ (d) $ab+ac$
16. $a-a=$ ____
    (a) *0* (b) 1 (c) $a$ (d) none of these.
17. $a \div a=$ ____
    (a) *0* (b) *1* (c) $a$ (d) none of these.
18. $a^3=$ ____
    (a) $aa$ (b) $a+a$ (c) $a+a+a$ (d) $a(aa)$
19. $a \div 0=$ ____
    (a) $a$ (b) 0 (c) 1 (d) ***none of these.***
20. $2x-3=0$ (mod 7) $x=$ ____
    (a) 0 (b) *5* (c) 1 (d) 7

| + | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 0 |
| 2 | 2 | 3 | 4 | 5 | 6 | 0 | 1 |
| 3 | 3 | 4 | 5 | 6 | 0 | 1 | 2 |
| 4 | 4 | 5 | 6 | 0 | 1 | 2 | 3 |
| 5 | 5 | 6 | 0 | 1 | 2 | 3 | 4 |
| 6 | 6 | 0 | 1 | 2 | 3 | 4 | 5 |

ADDITION TABLE
MODULO 7

(*See next page for Multiplication Table*)

| × | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| 2 | 0 | 2 | 4 | 6 | 1 | 3 | 5 |
| 3 | 0 | 3 | 6 | 2 | 5 | 1 | 4 |
| 4 | 0 | 4 | 1 | 5 | 2 | 6 | 3 |
| 5 | 0 | 5 | 3 | 1 | 6 | 4 | 2 |
| 6 | 0 | 6 | 5 | 4 | 3 | 2 | 1 |

MULTIPLICATION TABLE

MODULO 7

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## NEW KODAK PHYSICS BUILDING TO HAVE 160-FOOT LENS TUNNEL

A 160-foot lens tunnel is one of many special research features being built into a new 8-story physics building at Kodak Research Laboratories in Rochester, N. Y.

The tunnel will be used in research on lenses and optical systems. Its length is essential because in testing a new lens design, an observer or instruments must be far enough from the light source to have it appear as a single point of light.

The tunnel is insulated and covered with three feet of earth to keep interfering heat waves to a minimum. The tunnel is located at basement level, outside the building proper, so it will be free of vibrations from street traffic that might affect delicate measuring instruments.

Another anti-vibration provision is that heating and ventilating motors will be kept to minimum size. There are 16 fan rooms throughout the building, each employing a relatively small, vibration-free motor.

An "optical penthouse" on the roof of the building will provide for study of such subjects as long-distance photography and the physical characteristics of sunlight. A railed, tiled area for outdoor experiments will also be located on the roof.

Basement x-ray rooms with 24-inch concrete walls will permit the use of higher voltage x-ray equipment than that now employed in the Research Laboratories. All utilities to the x-ray rooms will be supplied underground to prohibit radiation leaks through air ducts or pipe shafts.

Especially important for electronics studies is a series of perfect electrical grounds. Already installed, the grounds consist of 3-inch copper pipes sunk into bedrock outside the concrete footings. Low resistance gound cable extends to the electronics laboratories from the ground pipes. The tubes may be filled with treated water for a perfect connection to ground.

## Projects for Greater Learning

Gerald Scrivens
*Titusville Junior High, Titusville, Pennsylvania*

Student projects in the general science classes of the Titusville Junior High School have proved to be very beneficial. The use of projects has broadened the scope of the course and given the student a chance to express himself in a way not ordinarily done in the classroom.

The project method of teaching is certainly not new. Learning by doing—the Dewey philosophy—has been practiced by teachers for many years. The ideas and methods used in project learning have and will continue to change. A project should be defined here as an "individual effort of the student." This may be in the form of a research report, model making, collections, or statistical analysis of some scientific endeavor.

I have been using the project method for the past ten years. Being an agriculture education graduate, perhaps projects were well instilled in me while teaching farm boys. The carryover from agriculture to general science was both easy and practical, however, and I believe that any teacher in any science subject field will find it easy to create motivation and interest in projects.

Certainly, general science, because of its wide range of subject matter, lends itself beautifully to student projects. Because of its ever widening scope, i.e., new developments in research, inventions and outer space, general science becomes increasingly more difficult to teach. The reason for this is quite evident—the year does not get longer, but the subject matter does.

Certain facts, demonstrations and experiments must be taught to a class group by the teacher. However, there are a great many opportunities for the student to do creative individual work outside of regular class work. The following procedure is one which I find very successful in the Titusville Junior High School.

We have an average enrollment of 180 students in ninth grade general science, about 30 per class—six times a day. With only one teacher for this large group, individual instruction is almost *nil*.

I use one class period in the beginning of each six weeks to explain the area in which I wish the student to do a project. For instance, the second day of school in September I described the procedure for making a student herbarium. I gave the student his choice of one of three catagories: cultivated flowers, wild herbaceous plants, and trees and shrubs. I then explained methods of collecting, drying, and mounting. I also mandated a minimum of 30 specimens. The rest was up to the

student. A deadline, approximately one week before report cards came out, was set when projects had to be turned in to me. The results of such a project were fabulous. From this project the student learned proper methods of collecting scientific specimens, preparing and preserving them, and properly marking and identifying each specimen.

The second six-weeks project was similiar to the first. However, this collection was left to the student's choice, as long as it was in the interest of science. Projects turned in consisted of wood samples, rocks and minerals, bark from trees, bird skins, insects and many others too numerous to mention. From the second six weeks work I could learn if the student understood what makes a scientific collection, that is, was it identified? Were there adequate specimens enough to call a collection? Did the student understand the accurate and orderly arrangement so necessary in science? The results showed that most of my students did acquire this knowledge.

The third six-weeks project will be in the field of electricity—building motors, telegraph keys, generators, wiring lamps, etc. In the fourth, fifth, and sixth six-weeks' projects we will cover the fields of astronomy and space travel, nutrition and health, and home improvement.

The above projects are all outside of class work. In fact, such learning procedures would be very difficult to put across in the class room. These projects have instilled in my students a kind of scientific curiosity that they otherwise would not have acquired.

I advocate these outside projects very strongly in any course where the teacher finds that his time is at a premium. I also believe that, at the junior high school level, projects such as the ones described above are a necessary part of the junior high school student's curriculum. They help to give him a sense of accomplishment and pride in something which he has done. If for no other reason it will at least draw him away from the television set for a few hours.

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## URGE SPACE PROBE TO TEST FOR POSSIBLE MARTIAN LIFE

A space probe passing within some 600,000 miles of Mars might show if life exists on the planet.

The probe could scan Mars, sending back to earth information about the spectra in the three to seven micron region, two California Institute of Technology scientists reported to the first International Space Science Symposium here. The spectra would be correlated with the visual light and dark areas of Mars.

Even if not more than a few thousand "bits" of information were transmitted to earth, the experiment would be "significant."

This is because the earth's atmosphere blocks most of the infrared light in the spectrum band between one and 100 microns. Some breaks in this atmospheric block have already permitted detection of what may be carbon-hydrogen bond molecules on Mars, indicating the existence of organic life.

## Student Reactions to Semimicro Chemistry Laboratory

**Carl Engels**

***University School, Western Michigan University, Kalamazoo, Mich.***

In 1956 and 1957 there appeared in the literature a number of articles on the use of semimicro chemistry laboratory methods in high school. Science teachers' conventions began to feature demonstration lectures on the technique. Laboratory furniture manufacturers began to include in their advertising statements to the effect that their furniture had been adapted to handle this kind of laboratory work. This fall one manufacturer[1] is offering a package arrangement on an entirely new line of laboratory tables for semimicro, which can be purchased along with all the apparatus to equip a laboratory completely. The apparatus is supplied by a company[2] which has specialized in semimicro equipment.

The semimicro method uses apparatus and glassware intermediate in size between the "macro" size (standard size used in most high schools), and the "micro" size (extremely small size used in advanced college courses). For example, the average size beaker used in the "semimicro" method is 50 ml., in the "macro" method this beaker would be 250 ml.

It is not necessary for the instructor to have special training in the procedures of Semimicro chemistry in order to introduce the method in his (or her) classes. One can become familiar with the simple techniques involved in a few hours. The major differences involved are as follows: (1) Instead of filtering, precipitates are separated by centrifuging, which is a faster and more practical method with the small quantities involved; and (2) reactions and evaporations are frequently carried out in drop quantities on microscope slides.

Students work individually, and have their own drawer or tote tray of apparatus and glassware. They have ready access to a tray of reagents, which is usually shared by two to a maximum of four students. This reagent tray varies in size; but the most popular one contains 107 bottles, some of which are 8 ml. in size and others 15 ml. in size. The 107 bottles in the reagent tray contain practically all of the solids and solutions the students will require during the school year. A wood block containing the five most commonly used acids and bases in 30 ml. dropping bottles is also provided on each student table. If the balances and centrifuges are advantageously located, traffic by students during the laboratory period can be virtually eliminated.

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[1] Kewaunee Manufacturing Company, Adrian, Michigan.

[2] Wilkens Anderson Company, Chicago, Illinois.

In the fall semester of 1957 the author explained to two of his chemistry students that the use of the semimicro chemistry laboratory was increasing rapidly and asked if they would be interested in trying it. They were eager to do so and ordered the apparatus and the *Weisbruch Semimicro Laboratory Manual.* After using the method for the remainder of the year, they were very enthusiastic about it and their experience was very helpful for setting up an 18-student laboratory for the following year.

A class of 36 students was set up to try the new method. Two teachers handled the lecture demonstration part of the course by teaching the whole class alternately. The same teachers handled the lab work. The first semester, half of the class used the regular macro method of laboratory work and the other half used semimicro. After working this way for a semester, the groups were changed to the other kind of laboratory work so that each student in the class would have both experiences.

The laboratory for macro work was a regular college-type general chemistry lab with long, 8-student tables and all services available, including individual hoods. The students worked in pairs under one of the instructors. The other half of the class, using semimicro, worked in a physics laboratory on tables 36 inches high. Only gas and electricity were available, so a plastic refrigerator dish served as sink or water supply or pneumatic trough. Each student had his own water supply in an 8-ounce polyethylene wash bottle. There was one sink in the room. Individual student apparatus was provided and stored in plastic tote trays, and a cabinet was provided to store the trays. Reagent trays and acid reagent blocks remained on the physics tables and did not interfere with physics laboratory work.

These conditions were far from ideal, but we were convinced that with this system good chemistry laboratory work could be carried out in any room which had in it tables and one sink. At the end of the school year, we asked each member of the class to fill out a questionnaire about his experience in the chemistry laboratory methods.

To the question, "Which of the two methods of lab work did you prefer?" 23 students preferred Semimicro, 10 preferred macro, and 3 had no preference. Reasons for their selection of semimicro were: smaller equipment is easier to use; it is easier to locate reagents; more can be done in a period; it is neater; little moving about is required; one gets more teacher contact; one must develop more accuracy; and better results are obtained. People who preferred the macro method said: results can be more easily seen; it is easier to work with larger apparatus; and the burners are not hot enough in semimicro.

To the question, "Did you prefer working alone or with a lab

partner?" 22 said with a partner and 14 said alone. Yet when asked whether they thought they learned more working alone or with a partner, 20 thought they learned more working alone and 16 spoke for the partner arrangement. The next question asked, "With which method do you think you could get more done in a lab period?" 27 students indicated the semimicro method and 9 thought the macro method. Since in the semimicro method they worked at high tables but had stools on which they could work sitting down, we were interested in knowing which way they preferred to work. Twenty-one liked working while standing up, 10 while sitting, and 5 said it was nice to be able to do both. When lab work was being done, most of the students were observed to be standing. Since the physics lab was without the common services of water, sinks, and hoods, we asked a question about which of these was missed most. Thirty-three felt that it was a hardship to be without sinks, and 3 felt hoods were needed. Nineteen students thought the instructor was most available to them in semimicro, 14 said macro, and 3 said there was no difference. To a question which asked which method allowed longer periods of work uninterrupted by moving about, 32 said semimicro, and 4 said macro. Upon being asked to list the most important advantages of the semimicro method, most students mentioned easy availability of materials and better, faster methods.

During the present school year we are using the semimicro method in one of three chemistry classes and will attempt to determine whether the students using it are more successful than those using the macro method.

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## NEW TABLE SHOWS HOURLY METEOR RATES

A new table that shows the number of "shooting stars" a trained observer can expect to see during any hour of darkness for any night of the year is now available.

The number varies from zero at 6:00 P.M., local time, on Jan. 1 to 66 at 3:00 A.M. on Aug. 11, when the Perseid meteor shower is at its height, to five at 7:00 P.M. on Dec. 31. The table was compiled by Dr. Charles P. Olivier of the University of Pennsylvania's Flower and Cook Observatory at Philadelphia.

The rates he computed from reports by many hundreds of observers are those that might be seen by a skilled observer at a good station on a moonless, clear night. Since conditions usually fall short of this ideal, the number of meteors actually observed will usually be smaller than shown in the table.

Dr. Olivier also calculated that during the 58-year period on which the table is based, a total of about 294,000 visible meteors could have been seen. The catalogue of hourly meteor rates, published by the Smithsonian Institution Astrophysical Observatory is available from the U. S. Government Printing Office here for fifteen cents.

# Modern Curriculum Problems in the Physical Sciences

Gerald Osborn (Moderator of Panel)
*Western Michigan University, Kalamazoo, Michigan*

This is a partial report of a panel discussion at the 1959 Central Association of Science and Mathematics Teachers Convention in Chicago. The panelists were:

Mrs. Jacqueline Mallinson, Consultant in the Teaching of Elementary Sciences, Kalamazoo, Michigan.

Mr. Norman Rubel, Lakeview Public Schools, Battle Creek, Michigan who discussed the problems from the standpoint of a Junior High Teacher

Mr. Gordon Noble, Central High School, Kalamazoo, Michigan gave consideration to Chemistry Curriculum problems.

Mr. Loyal Phares, W. K. Kellogg Community College, Battle Creek, Michigan, presented curriculum problems of the Junior College in the Physical Sciences.

The following questions were considered by the panel.

A. Early Elementary.
1. What are some of the greatest weaknesses in our present elementary science program?
2. Do you feel that modern school administrators are cognizant of any shortcomings in our elementary science program?
3. What are some of the curricular changes of grades kindergarten through nine that you feel should be made for the improvement of science education?
4. Do you feel that there should be a definite science sequence for the elementary grades?
5. If so, what sequence would you suggest?
6. What should be the nature of the laboratory experiences at each grade level?
7. What do you consider to be the main objectives of elementary science?

B. Junior High School.
1. What is the main goal of the junior high science program?
2. In what ways do you feel the science curriculum in the junior high school should be altered to obtain such a goal?
3. What are some of the weaknesses of our present junior high science courses?
4. What should be the nature of the junior high laboratory experience in science?
5. How can a science teacher keep up-to-date on recent developments and discoveries?

C. Senior High School.
1. What should be a sound, up-to-date philosophy of science teaching? How would this affect curricular change?
2. What are the advantages and disadvantages to special ability grouping in the physical sciences at the secondary school level?
3. What should be the content of a modern chemistry course at the secondary school level?
4. How should mathematics and science be related in the modern curriculum?
5. What emphasis should be placed on lecture-demonstrations in secondary school science?
6. How should the laboratory work be properly correlated with the class work?
7. What are some of the new developments in high school chemistry? In high school physics?
8. What is your opinion of the new high school program in physics as outlined by the M.I.T. study? I refer to the work of the physical science study committee stimulated primarily by Dr. Jerald R. Zacharias of M.I.T. How would such a program fit into your high school?

D. Junior College.
1. What are some of the main goals a junior college teacher would wish to attain with his physical science program?
2. What should be the high school preparation of the junior college freshman
   (a) in mathematics?
   (b) in chemistry?
   (c) in physics?
3. What type of laboratory work should be done in the junior college? Where should the emphasis be placed?
4. What do you feel would be some desirable curricular changes in the 1st and 2nd year of college physical science?

At this time we are presenting papers from two of the panelists. Perhaps the others may come at a later date.

## The Role of Elementary-School Science in the Modern Curriculum*

Jacqueline Mallinson
*Kalamazoo, Michigan*

Since October 4, 1957, there has been much discussion and debate

* An abstract of a report presented at the Annual Convention of the Central Association of Science and Mathematics Teachers, Chicago, Illinois, November 27, 1959.

concerning the quality of instruction and achievement in all fields of education. The area of science education in particular has received a great amount of attention. Many of the criticisms and comments aimed at science education in the American schools have been unjustified and indefensible in the light of research findings. Nevertheless, the sudden focus of attention on this area has motivated a scrutiny of science programs in the schools. The resulting evaluations, if properly conducted, can be most valuable.

Recent articles in newspapers, professional journals and popular magazines have been concerned primarily with science programs in secondary schools and colleges. While such an emphasis may be justified, this report will be concerned only with the problems related to the program of science education in the elementary schools.

Of all the areas taught in the elementary school, science is probably the newest and most "immature" of all of them. Because of its immaturity, unlike other subject-matter areas that developed from the bottom, science developed from "top to bottom." The earliest courses in science offered at the pre-college level were in the secondary schools. These included chemistry, physics, botany and zoology. Such courses were taught in most of the private secondary schools as early as the 1800's. They were designed chiefly as miniature editions of the analogous college courses and were for those students planning on college training.

Science did not "grow down" to the junior-high-school level until the early 1900's, when the nature of the secondary school population changed markedly. At that time science programs were "pushed down" still farther, now entering the pre-high school grades. During the two decades that followed, junior-high-school general science was the first formal science training offered to public school students.

The first authoritative statement concerning the placement of science in the elementary grades did not appear until 1932. This statement appeared in the Thirty-first Yearbook of the National Society for the Study of Education.[1] For the first time, it was suggested that science training should begin in kindergarten and continue through grade 12.

Despite this statement declaring the importance of science in the elementary-school program, the movement "took hold" slowly. There are undoubtedly a number of reasons for the slow growth of elementary science. They include such frequently-listed problems as lack of science training on the parts of elementary teachers; lack of time in the school day for science; and lack of science equipment. While these problems, in the opinion of the writer, may have some merit,

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[1] *A Program for Teaching Science.* Thirty-first Yearbook of the National Society for the Study of Education. Part I. Chicago: Distributed by the University of Chicago Press, 1932. Pp. xii+364.

the difficulties hampering the development of elementary science programs are far more basic. They are (1) lack of direction, (2) lack of continuity, and (3) lack of order.

Of these three difficulties, lack of direction is probably the most pressing problem. The lack of direction is evidenced by the fact that elementary teachers cannot articulate in "words of one syllable" *what* a program of elementary science is designed to accomplish. As a result, elementary programs have been "hit or miss" and have been characterized, in many cases, by fragmentary, disconnected presentations of the classic areas of science. In the main, these presentations have been designed to impart some scientific information to the child, but fail to assist him in being a better performer.

Since the launching of Sputnik I in the fall of 1957, there has been a frantic effort on the part of most elementary-school administrators to initiate a science program, if one did not already exist in their schools. However, in spite of the sincere efforts of curriculum workers, there still appears to be much confusion concerning what constitutes a *good* program of elementary science. It is likely that one of the main reasons is this lack of tangible goals of instruction.

The author does not claim to offer an entirely new set of goals of instruction. However, it does appear that there are some specific abilities that all students should acquire as a result of studying science. Further, it is suggested that these abilities can be stated in terms of "one-syllable" that all elementary teachers can understand. If the list is accepted, it is hoped that it may assist the elementary teacher in developing a meaningful program of elementary science that has, in addition, order and direction. The suggested list of goals, or "doings" of science follows:

1. The ability to observe the objects that exist, and the phenomena that take place, in the child's environment and to report accurately what he observes. In other words, the student should be taught to describe.
2. The ability to compare objects and phenomena with respect to their (a) likenesses and (b) differences. These two skills are different from one another.
3. The ability to rank information in terms of its relative importance.
4. The ability to determine whether or not there is enough information available to warrant making a conclusive or even a tentative answer. In other words, to decide how much of an answer can be drawn from the information available.
5. The ability to determine what *kind* of information is still needed in order to formulate an answer to a question, or locate data for a problem.
6. The ability to decide on the most efficient way to obtain the needed information to answer a question or problem.
7. The ability to carry out an experiment with the materials available, if it is decided that an experiment is the best way to obtain the answer to a problem.

Obviously, no single science experience can contribute to all these

skills. Some will contribute to only one, others to several. Further, all children cannot be expected to become equally proficient at all skills. The development of the skills will, of course, depend on the maturity of the child.

To this point, no mention has been made of the role of subject matter in the development of an elementary science program. It is obvious that there are certain traditional areas of science that should be included in a well-balanced elementary-school program. However, rather than merely presenting fragmented subject-matter topics, designed to teach children a few isolated facts, the traditional areas should be used as the vehicles by which the skills listed above are developed in children.

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## Modern Curriculum Problems in the Physical Sciences*

### A Discussion of a Special Chemistry Course

Gordon Noble

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Every citizen today surely needs to know something about science. He doesn't need to remember any particular formula, fact, law or theory, but he should know what science is for numerous reasons.

Our students live in an age of science. In the modern curriculum there should be a laboratory course in one of the physical sciences required of every secondary school student. Since we hope that at least some of them will contribute to the advancement of science, it is a matter of intense concern that they have a special kind of curriculum in the physical sciences. One such attempt is a special course in chemistry which is being offered to certain selected students during the school year 1959–60 at Kalamazoo Central High School.

Students were selected for the course on the basis of interest, aptitude, achievement and special training in mathematics. Emphasis in the course is placed on theories, basic assumptions and relationships to other science and mathematics.

A description of the course runs as follows:

Introduction:

The course is conceived as an introductory general chemistry which deals with the basic assumptions and logical structure of chemistry.

Objectives:

1. Examine the fundamental theories and concepts of modern

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* This outline is based on the scope and sequence of the textbook, *Modern Chemistry*, by Dull, Metcalfe and Williams, Henry Holt and Co., 1958, which is the basic text for the course.

chemistry using atomic structure and chemical bonding as the central theme.

2. Promote critical, independent thinking about matter and its transformations.
3. Develop an understanding and appreciation of science, the scientific method and the universe.
4. Develop the ability to draw conclusions from accumulated data.
5. Develop a competency in and understanding of the mathematics used in chemical calculations with a justifiable self-confidence in these fields.

General Procedures:

A *systematic* approach to the study of general chemistry characterizes this course. The approach de-emphasizes the detail and minutiae of descriptive material and places emphasis upon the big ideas and important principles of chemistry. More time is thus allowed for demonstrations, laboratory work, and a thorough consideration of theoretical concepts. A logical integration with physics receives utmost consideration. A logical arrangement in the organization of the course content leads one major idea into another. Emphasis is not primarily placed on recognition and recall but on an understanding of important concepts.

### Content

I. What role does science play in a changing world?
   1. Science and the citizen
      a. What is science?
      b. What is the scientific method?
      c. Does science create social problems?
      d. Does science solve social problems?
      e. The language of science.
      f. The mathematics of science-conversion factors and the metric system.
   2. Chemistry—a modern science
      a. Beginnings of modern chemistry.
      b. Contributions of the Ancients.
      c. Alchemy and the Phlogiston theory.
      d. Work of Robert Boyle.

II. What is the nature and composition of matter?
   1. The physical state of matter
      a. Kinds of matter.
      b. Materials and substances—elements, compounds, mixtures.
      c. Introduction to the concept of energy change, mass-energy relationship.
   2. How matter undergoes change
      a. Physical and chemical changes.
      b. The law of definite proportions.
      c. Atomic weights.
      d. The emergence of modern atomic theory.
      e. Evidence for sub-atomic particles—proton, neutron, and electron.
      f. The periodic law.

3. Molecules and valence
   a. Molecules and molecular weights.
   b. Valence-combining capacity.
   c. The law of multiple proportions.
4. Introduction to the chemical bond
   a. Origin and properties of ions.
   b. Electrovalence.
   c. Covalence and coordinate covalence.
   d. Structure of crystals.
   e. Hydrogen bonding.

III. How do theories explain chemical and physical phenomena?
1. Chemical changes
   a. History of oxygen and hydrogen and water.
   b. Introduction to oxidation-reduction.
2. The gas laws
   a. Volume-pressure-temperature relationships.
   b. Kinetic molecular theory.
3. Water and solution
   a. Physical and chemical properties of water.
   b. Nature and characteristics of solution.
4. A philosophical consideration of the nature of laws, theories, truth and change and their applications to chemistry and science

IV. How is mathematics used in chemistry?
1. Chemical calculations
   a. Mathematical theory, system and operation as applied to the exact sciences.
   b. Chemical symbols and formulas.
   c. Molecular weights—mole concept—percentage composition.
   d. Chemical equations and energy of reactions.
   e. The equation as a weight relationship.
   f. Molecular composition of gases.
   g. Volume relations in chemical reactions.

V. What is the nature of carbon and its simple compounds?
1. Carbon
   a. Physical properties.
   b. Chemical properties.
2. The oxides of carbon
3. Hydrocarbons
   a. Aliphatic hydrocarbons.
   b. Aromatic hydrocarbons.

VI. How does the presence of ions help us understand chemical and physical phenomena?
1. Ionization
   a. Formal, molar, normal and molal solutions and mole fraction.
   b. Conductors and electrolytes.
   c. Theory of Arrhenius.
   d. Electrolysis and oxidation-reduction.
   e. Current theoretical considerations of ionization.
2. Acids, bases and salts
   a. Ionic concept of acids and bases.
   b. Evolution of acid-base theory.
   c. Neutralization and salt formation.
   d. Hydrolysis.
   e. Bronsted concept and applications to acid-base reactions including hydrolysis.
   f. Equivalent weights.

VII. What is the nature of chemical reaction?
1. Chemical equilibrium
   a. Reversible reactions.
   b. Factors affecting the speeds of reactions.
   c. Law of mass action.
   d. The equilibrium constant.
   e. Le Chatelier's principle.
   f. Common ion effect.
   g. Ionization constants.
   h. Hydrolysis.
2. Reactions that go to an end
   a. Gas formation.
   b. Precipitate formation.
   c. Slight ionization.

VIII. What is the sub-atomic nature of matter and its applications?
1. Electrons, protons, neutrons—sub-atomic particles
   a. Historical development.
   b. Conductivity through gases.
   c. Nuclear research instruments.
   d. Unit charges.
   e. The nature of radioactivity.
   f. Atomic number, mass number.
   g. Lewis-Langmuir concept of the atom.
   h. Current concepts of atomic structure.
   i. Discovery of the neutron.
   j. Artificial radioactivity.
   k. Nuclear reactions.
   l. Radioisotopes.
   m. Nuclear energy—the philosophical consideration of its beneficial and destructive effects.

IX. How do we use the periodic table in a systematic study of chemistry?
1. The active metals
   a. The sodium family.
   b. The calcium family.
2. The halogens and sulfur
   a. The halogen family.
   b. Sulfur and sulfides.
   c. The oxides and acids of sulfur.
3. The nitrogen family
   a. The atmosphere—quantitative determination of nitrogen and oxygen in air.
   b. The inert gases.
   c. Nitrogen and its compounds.
   d. Phosphorus, arsenic, antimony and bismuth.
4. The light metals
   a. Beryllium, magnesium, aluminum, and titanium.
5. The heavy metals
   a. The iron family.
   b. The copper family.
   c. Zinc, cadmium, mercury, tin and lead.

X. What is the colloidal state?
1. Colloidal suspensions
   a. Colloidal size.
   b. Surface chemistry.
   c. Adsorption and its applications—chromatography.
   d. Contact catalysis.

e. Suspensoids.
f. Emulsoids.

XI. How does the study of organic chemistry affect our daily lives?
1. Common organic compounds
a. Fuels.
b. Petroleum.
2. Chemistry of organic compounds
a. Electronic configuration of the carbon atom.
b. Basic concepts of organic compounds.
c. Aliphatics and aromatic hydrocarbons.
d. Alcohols.
e. Ethers.
f. Aldehydes and ketones.
g. Organic acids.
h. Esters, fats and oils.
i. Amides and amines.
j. Chemistry of living cells.
k. Synthetic organic chemistry.

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## AMERICAN INSTITUTE OF BIOLOGICAL SCIENCES TRANSLATION PROGRAM

The American Institute of Biological Sciences is currently translating and publishing seven Russian research journals in biology. These journals are translated with support from the National Science Foundation, which is eager that such information be more widely distributed to biologists throughout the world. It is hoped that this material will aid biologists in research, prevent duplication of work, give some idea of the work being done by Soviet scientists in the field of biology, and also bring about a better international understanding among scientists.

Because of the support of the National Science Foundation, the AIBS can offer these translations at a fraction of their publication cost, with even further price reduction to AIBS members and to academic and non-profit libraries. This reduction, the AIBS feels, places the translation within the reach of all biologists.

The journals currently being translated are: Doklady: Biological Sciences Section; Doklady: Botanical Sciences Section; Doklady: Biochemistry Section; Plant Physiology; Microbiology; Soviet Soil Science; and Entomological Review.

In addition to its program of Russian Biological Journal translations, the AIBS has instituted a separate program of translation and publication of selected Russian Monographs in biology.

It was felt that the program of Journal translations was not sufficient to cover all of the significant work being done in all fields of biology by Russian scientists. With the aid of competent authorities, the AIBS has translated and published six Russian monographs and one monograph is in the process of being published. In addition, several prominent monographs in various biological areas are being considered by the AIBS and the National Science Foundation for translation and publication. The monographs that have been published are: Origins of Angiospermous Plants by A. L. Takhtajan; Problems in the Classification of Antagonists of Actinomycetes by G. F. Gauze; Marine Biology, Trudi Institute of Oceanology, Vol. XX, edited by B. N. Nikitin; Arachmoidea by A. A. Zakhvatkin; and Arachnida by B. I. Pomerantzev. The manuscript for Plants and X rays by L. P. Breslavets is in the final stages of preparation and should be published early in 1960.

Additional information pertaining to this program may be obtained by writing to the American Institute of Biological Sciences, 2000 P Street, N. W., Washington 6, D. C., U. S. A.

# Some Needed Breakthroughs in Mathematics Education

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The purpose of this article is to point out certain breakthroughs which are needed in mathematics education. A few of these important breakthroughs have appeared in both teaching methods and courses of study, but need to be further developed and refined. Other breakthroughs are needed in order that better programs in mathematics be assured for pupils. Along with these challenges in mathematics education, a possible plan of action for schools and teachers will be presented.

## I. Instructional Materials

There must be a breakthrough in the types and uses of instructional materials and classroom facilities for the teaching of mathematics. No longer can the mathematics teacher be expected to do the job with only the aid of the blackboard, chalk, and textbook. The idea of the mathematics classroom as a "learning laboratory" must be fully realized; possessing all various types of audio-visual equipment and other instructional aids valuable for facilitating and strengthening learning in mathematics.

Through the National Defense Education Act, many schools throughout the nation have obtained mathematics materials that have been recognized as valuable in the teaching of mathematics. To date, however, the thinking as to what is needed in a learning laboratory for mathematics has been rather shortsighted. Materials for the classroom in the majority of cases have been confined to the more common devices or instructional materials such as the compass, protractor, geometric solids, coordinate charts, and others of this nature. Although some of the audio-visual materials of this type will no doubt aid pupils and teachers in mathematics, these materials hardly will solve the existing problems and meet obvious challenges facing mathematics education.

New developments in mathematical content, changes in teaching methods, providing for all levels of ability, introduction of new courses, and the demands for providing quality programs at all levels will make it increasingly difficult for teachers to provide even a minimum program for all pupils unless constructive changes are realized. The individual teacher who meets from 100–175 pupils each day cannot hope to serve adequately the pupils in her mathematics classes without help of excellent instructional materials and cooperation from other staff members. A needed breakthrough in this area of

mathematics education must be in terms of providing the teacher with adequate materials of instruction which will make possible a dynamic classroom-laboratory for the teaching and learning of mathematics.

*A Plan of Action for the School*—Schools can meet the challenge for the improvement of instructional facilities and materials in several ways. Flexibility in arrangement and use of the classroom space appears essential. For example, moveable wall partitions in the classroom can make possible the carrying out of varied activities simultaneously in the same unit of space. In one section of the room listening stations provided through electronics for pupils to use in receiving additional practice and/or explanation in a specific area of study might be included.

Libraries of records and prepared tapes available for use might include enrichment work for pupils or provide answers and explanations to lessons corresponding to the work of the particular class. Thus, pupils may use pre-recorded materials as needed on an individual basis which allows additional time for the teacher to work with smaller groups of pupils or to direct extension or remedial activities in other parts of the room. In another section of the room, partitioning makes possible the viewing of films and filmstrips which supplement the ongoing activities in the unit of work.

Emphasis is given to the use of self-evaluation; pupils become aware that they cannot expect the teacher to be on hand at all times to tell them the next step or to provide the answer to each problem. More independent work and checking of progress by the individual are made possible through a variety of resource materials and devices. The audio-visual or production center within the building serves an important role in helping the teaching staff with equipment and materials needed for effective instruction in mathematics. Portable libraries including pupil references and supplementary textbooks that can be transported into the classroom for use in a specific unit of work and removed with little difficulty when no longer needed would be of considerable value. At the same time, library work should be increased for pupils capable of doing advanced work and research on an independent basis.

Other steps by the school which would be of help in meeting the challenge for a needed breakthrough for improved facilities and instructional materials in mathematics education might include the following:

1. An evaluation of the present program in mathematics and the determination of materials and equipment needed in order to make improvements and progress should be carried out cooperatively. Beginning on a modest scale, schools should plan for a continuous improvement of instructional facilities and materials.

2. Provision for television classes or viewing on other basis might be utilized by both the large and small school. Lessons presented by area specialists in mathematics can be enriched and extended in the classroom by the teacher who is working cooperatively with the studio teacher. It would not seem too optimistic to visualize pupils viewing and reviewing mathematics lessons through the facilities of video-tape. Such a technique makes the scheduling of pupils' work in mathematics more individual and flexible.
3. Proper preparation for the use of instructional materials demands teacher time and effort. In providing quality instruction for pupils, administrators must give full thought and attention to the problem of teacher load. In determining the teacher load, all factors which make up the total load of the teacher must be considered.

## II. Curriculum in Mathematics

Another breakthrough needed in mathematics is in the area of curriculum. An emphasis on the teaching of mathematics as a logical science with special attention to the structure of mathematics is essential. The change in approach from that of memorizing certain rules and mechanical manipulation with the thought of "applying these rules later" must change to a point of view where the challenge is in the mathematics itself. In the elementary grades, the basis for the logical and structural approach to mathematics must be developed.

The nature and development of a number system, use of various number bases, and the rationale in the use and role of the mathematical laws need to be given special attention. Recent national studies have indicated that certain topics in mathematics can be introduced at lower levels than previously thought possible.

Grades seven and eight can reduce the work in social applications of mathematics and provide content and instruction which extends concepts of structure of mathematics which will develop a stronger foundation for future work in algebra and higher mathematics. Approaches to elementary work in algebra should develop the topics from a structural point of view with the development of an understanding of the concept of a mathematical model; the definitions and assumptions made at the outset determining the nature of the "model."

Geometry must continue to provide avenues of study of a logical science with more attention given to the reasoning and the nature of proof. For some pupils, work in trigonometry and analytics, which perhaps will be integrated, can be provided, but acceleration beyond this point in courses in mathematics in the high school does not seem warranted.

*A Plan of Action for the School*—Schools should re-examine present offerings in terms of recent research and needs in mathematics educa-

tion. With staff members, principals, coordinators, and consultants working together, the school can develop constructive approaches. The development of curriculum guides in specific areas of mathematics which set forth objectives, establish course content, and suggest experiences and activities for pupils can be done cooperatively. The search for resource materials which will be of value in the accomplishment of set goals then can be determined.

Such a plan of action involves further professional growth on the part of each teacher. The needed breakthrough for professional growth on the part of mathematics teachers is discussed later in this article.

### III. Team Teaching in Mathematics

The dynamic classroom unit and up-grading of instruction implies the need for quality instruction for pupils. It is becoming increasingly apparent that teacher talents must be utilized more efficiently. Teams of teachers and other staff members must work together, coordinate efforts, and cooperatively plan the experiences to be provided for pupils. A breakthrough is needed in this area of mathematics education.

This approach involves teams of teachers working to utilize individual talents in the place and at the time they will do the most in providing quality instruction. Basic instruction in a specific area of mathematics should be given by the most qualified person available; some presentations provided for groups of 100–300 pupils. Competent teachers who are also important members of the team provide adequate help for the "teacher specialist." The materials center in the building, library staff, reading coordinator, and other team members serve as important resource persons in this approach to mathematics teaching.

*A Plan of Action for the School*—Schools can take steps to develop cooperative efforts toward a better mathematics program in several ways. Many of the suggestions presented earlier in this article would prove of value in meeting this challenge. Other considerations in this area include the following:

1. Schools can work to provide a more flexible scheduling procedure to facilitate the use of community resources involving mathematics and its use; flexibility also permitting more independent work and study by indivisuals.
2. Provide in the regular school budget for obvious expenses involved in developing the plan of action determined by teachers and administration.
3. Encourage the use of new methods, experimentation, and team action on the part of all teachers. Develop long-range plans directed toward improvements of coordination of teachers' efforts and goals in mathematics.

## IV. Professional Growth for Teachers

All of the breakthroughs needed in mathematics education have implications for professional growth for teachers. Proper use of materials and equipment in the dynamic classroom will call for new learning and preparation on the part of the teacher. New approaches to teaching and developments in curriculum make in-service instruction of paramount importance. Opportunities for teachers to gain a knowledge of new approaches and new developments in mathematics are limited for the most part only by the time and personal initiative of the individual. It must be realized that teachers need time allotted for professional growth and that such time will pay dividends in terms of better programs for pupils.

*A Plan of Action for the School*—Although all of the various suggestions for providing continuous growth for teachers cannot be discussed, the following ideas are ones that schools might consider as means for meeting the challenge of the needed breakthrough in this area of mathematics education:

1. Improve supervisory services for teachers. Coordinators and supervisors are in the best position to give constructive help that teachers need in regard to finding new materials, uses of grouping and varied activities, and other phases of the program in mathematics.
2. Provide time on perhaps one or two afternoons per week on an alternate basis for teachers to meet in in-service sessions. In some instances, classes meeting during the same time of the day can be combined for instructional purposes while one or more teachers are released for study or formal class work.
3. Encourage teachers to apply for and to attend summer institutes in mathematics. Organize in-service programs which utilize consultants and persons who have attended such institutes for informing other teachers.
4. Make available to teachers good materials of national organizations, state organizations, and local groups. Encourage teachers to join professional mathematics groups at the state and national levels. Provide a professional library in the school which contains journals and references on mathematics and education.
5. Encourage departmental meetings of mathematics teachers. Provide opportunities for teachers to attend conferences, workshops, and programs on mathematics and mathematics teaching. On occasion, use highly qualified resource persons in the community to release teachers for such worthwhile activities.

## V. Other Needed Breakthroughs in Mathematics

Other needed breakthroughs in mathematics education which have definite implications for the school include the following:

1. Techniques of Evaluation—Electronic devices and other techniques must be utilized for evaluating the work of the pupils. Teachers should have an important part in the guidance and determination of objectives for the class and the individual; concern with the preparing and presenting of quality instruction and guidance of classroom activity are essential. Grading of daily paper work and similar tasks must be minimized.

2. Grouping—Grouping procedures and techniques must be continually improved and refined.
3. Public Relations—The public must be kept informed as to the role of mathematics in the lives of all persons; changes in methods and content and reasons for these developments should be clarified; and various media used to review the program in mathematics. In a recent state publication, a leading educator was quoted as stating that, "even though algebra never changes that some of the writing in textbooks varies from time to time." If a leading educator apparently is so misinformed, others including the layman certainly need more information on mathematics teaching and its goals today.

## Summary

Certainly all of the possible breakthroughs in mathematics education have not been exhausted in this article, but a few of the major challenges facing the school and some possible plans of action have been discussed. If the schools now and in the future are to meet the growing demands for a mathematically educated people and the provision for an optimal program in mathematics for all pupils, all persons in the school and the community must give serious thought and attention to the breakthroughs needed in mathematics education and their implications for improved programs in this area in schools throughout the nation.

## "HORN" THOUGHT TO PLAY IMPORTANT ROLE IN MEMORY

A horn-shaped segment of the brain may decide what is worthwhile remembering and help recall the information when situations demand the information.

One activity of this brain segment may be to "approve" an incoming message (or stimuli) for permanent deposit as a memory trace in an appropriate neural system. Furthermore the hippocampus may assist in recalling the item of information in a conditioned learning (goal-directed) situation.

Experiments showed that certain rhythmic waves from this brain segment provide a sensitive correlate of the animal's ability to engage in goal-directed performance. This involved training the animal to approach a food reward in a maze box.

When the animal responded as he had been trained, the rhythmic waves seemed to signal his correct approach to the reward. Drugs which caused the animal to forget his training for a time so that he wandered aimlessly about led to abrupt change in wave patterns. The rhythmic pattern returned when the animal again responsed as it had been trained.

This rhythmic pattern from the hippocampus probably helps a person drive to work each day over a familiar route. The unconscious cues that tell where to turn were probably indelibly inscribed in the brain with the help of the hippocampus. And as long as this rhythmic brain wave persists the driver will not make a wrong turn.

## LIGHTNING SUGGESTED AS TORNADO CAUSE

Lucretius, the Roman philosopher who died in 55 B.S., proposed that electrical energy might be a cause of tornadoes. Now Dr. Bernard Vonnegut of Arthur D. Little, Inc., Cambridge, Mass., in an issue of *Journal of Geophysical Research* proposes that this idea be investigated. He suggested that there might be enough energy in an electrical storm to power a tornado, and called for special research, including study of the accounts of eyewitnesses who recall lightning or luminous glows before, during or after tornadoes.

# The Place of Planetaria in Teaching Space Science*

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When it was learned by the speaker that the science teaching societies were planning a general session devoted to "Man and Space Travel," the thought of a symposium devoted to "Planetaria and their Use for Education" was still fresh in his mind. It seemed obvious that many of the papers† presented on this symposium, which was held at the Cranbrook Institute of Science, September 7–10, 1958, under the auspices of the National Science Foundation, had a direct application to the theme of this session.

This paper, then, will consist largely of a gleaning from these papers of information and ideas pertinent to the ways in which planetaria can be utilized by those concerned with teaching space science. As a general statement of the application of this device to this purpose, I quote from the preface to the above-mentioned symposium: "The swift-moving events of the space age have imparted a new urgency to the study of astronomy; the solar system and the universe have suddenly become areas of immediate concern to unprecedented numbers of laymen; and there is a growing demand for the services which the planetarium provides."

As regards the significance of the planetarium to teaching, I quote further from the same preface: "This Symposium contains abundant evidence to show that in the field of audio-visual aids there is perhaps no more fascinating or potent an educational device than the planetarium. At its best, the planetarium is a broadly informative, dramatic stimulus to learning; at the very least, it is an effective means of presenting certain astronomical concepts."

When used for public demonstrations, usually with a different theme each month, "the educational aspects of the planetarium are two-fold: the first is to stimulate interest in astronomy and related sciences; and the second, to increase the layman's understanding and knowledge of the universe."

Some idea of the subject matter which can be successfully demonstrated with the planetarium can be gained by a perusal of the monthly themes, as presented this past year at the Cranbrook Institute of Science. These themes were as follows:

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|---|---|
| January | TELLING TIME BY THE SKY |
| February | SOUTH WITH THE STARS |

* A paper presented at the Science Teaching Societies general session: Man and Space Travel, AAAS meeting, Chicago, Illinois, December 27, 1959.

† Planetaria and their Use for Education. Miriam Jagger, Editor. Bulletin 38, Cranbrook Institute of Science, 1959, Pp. 200.

| | |
|---|---|
| March | SPRING IN THE SKY |
| April | COMETS AND METEORS |
| May | EXPLORING THE MILKY WAY |
| June | THE SUN, OUR DAYTIME STAR |
| July | NORTH TO POLAR SKIES |
| August | CAPTIVES OF THE SUN (The Planets) |
| September | AUTUMN IN THE SKY |
| October | THE MOON, OUR NEAREST NEIGHBOR |
| November | COLOR IN THE SKY |
| December | STAR OF BETHLEHEM |

As previously indicated, to make these themes more effective, accessory projectors designed to create special effects were used in conjunction with the basic planetarium projector. As an example, for the program concerned with comets and meteors, a special meteor projector was designed and constructed which simulated meteor showers. During the program, "Color in the Sky," an ingenious projector was used to imitate the *aurora borealis*. For most of the programs, however, the planetarium projector itself is quite adequately designed for and equipped with devices to illustrate many astronomical phenomena and to teach basic astronomical concepts, The component parts of this instrument include a Star Projector, which produces the illusion of the stars and the Milky Way; a Solar System Projector that shows Sun, Moon and the visible planets—Mercury, Venus, Mars, Jupiter, and Saturn; the Geocentric Earth Projector, which gives an "inside-out" view of the earth; and a control console, from which all operations of the instrument, as well as lighting and musical accompaniment, are directed.

The Astronomical Triangle Projector, the Meridian Projector, and the Projection Sextant are all used in explaining basic astronomical and mathematical concepts that take on new meaning when illustrated by the Planetarium.

In addition each instrument comes equipped with a Twilight Projector, to create the effects of dawn and dusk; a Projection Pointer, which throws a sharply defined arrow of light on the dome to point out objects under discussion; Room Illumination Projectors to provide adequate light for entrance and exit as well as serving to aid adaptation to darkness; and a Satellite Projector to demonstrate the orbits of man-made satellites.

It is important to mention at this point that the public programs featuring a special topic always include an introductory demonstration of fundamental astronomy; in fact, the greater part of each demonstration is devoted to such fundamentals.

Perhaps the type of program which has attracted the most attention to the planetarium, and the one for which it has no rival, is that of simulated space trips. There is no place in the universe too far out

in space to reach in one's imagination within the confines of the planetarium. It is here that special effects, such as the count-down, the sound of rocket engines, views of the earth from outer space, the surface of the moon and other space phenomena, not only help but are essential to the illusion. It is important, however, to keep the demonstration within plausible bounds, it must "stick to the facts" and not go over too far into the realm of science fiction. In this connection, it is interesting to note that some demonstrations which were a few years ago thought of as being too fictitious are now being presented as a matter of fact.

One of the most significant developments in the use of the planetarium, which has come about as a result of the availability of a relatively inexpensive projector, is its installation in many schools, museums and other educational institutions which heretofor could not afford such a teaching aid. School systems in almost 200 communities now provide a singular experience through the planetarium from which both youngsters and adults can acquire a basic understanding of the sky so important to today's expanding curriculum.

Thus, the planetarium has moved from its role in producing "spectaculars" to a position of great importance in providing a basic knowledge of astronomy at all levels of education. "Neither a plaything nor a frill, it is as vital a part of educational equipment as a physics or chemistry laboratory."

The planetarium experience is one that appeals to children of all ages. In the early school years, the planetarium experience lays the foundation for broad intellectual curiosity and the desire for more knowledge. High school students find the planetarium experience an opportunity to broaden their horizons, literally and figuratively.

As a case in point, Ann Arbor High School in Ann Arbor, Michigan, was one of the first high schools to have a well equipped planetarium. The funds for the purchase of this instrument came largely through a gift from Argus Cameras, Inc., of Ann Arbor. In planning the new high school building, the installation of a planetarium was anticipated. As a consequence, a specific area in the building was set aside for this purpose.

From an organizational standpoint, the planetarium is a facility of the science department of the Ann Arbor High School. The presence of a planetarium within the high school building provides exciting educational possibilities. One which it immediately suggested was the desirability of offering a course in astronomy. This course, which is descriptive and non-mathematical, is open to 10th, 11th and 12th grade students. It is a one semester course which meets for one class period on alternate days. In addition to this specific course, the

planetarium has made it possible to strengthen the units of study in astronomy in both junior and senior high school science courses. It not only facilitates more effective instruction, but it is an unusually dynamic device for stimulating the interest of the student and motivating him to learn.

Enriched instruction has been made possible in several other high school courses because of the planetarium. Specifically, it has been used in courses in meteorology, air age, world geography and, in an incidental way, in solid geometry.

Another, and equally important, use of the planetarium in the high school has been to enrich the programs of some of the gifted science students. Several students have given planetarium demonstrations and lectures. One young man prepared and presented a Christmas program which he subsequently submitted as a project in the annual Westinghouse Science Talent Search for which he received an honorable mention. As part of this project, he built several accessory devices including a very effective *super nova* projector.

In the first three years of its operation, the planetarium played host to more than 17,000 visitors, not only students from its own and adjoining school districts, but from colleges, universities, civic groups, clubs, scouts, and church organizations.

At the college and university level, the planetarium—in courses beyond the introductory phase—can be of assistance in clarifying confusing aspects of spherical astronomy involved in the determination of time, stellar coordinates and geographic position. In introductory courses, where a great deal of time is devoted to the apparent motions of the celestial sphere and members of the solar system, and to the heliocentric interpretation of these motions, the planetarium has great potentialities. Special attachments to illustrate the visual appearance of such phenomena as the Milky Way, eclipses, proper motions and variable stars are better understood when visually presented through the use of the planetarium than by a descriptive lecture.

Another interesting application of the planetarium to higher education is its use in teaching navigation. One problem concerns the presentation of a perspective view of the celestial sphere and astronomical triangle, a view essential to the comprehension of celestial navigation. Another basic requirement for students of navigation is the ability to recognize both the stars and the constellations. The planetarium helps immeasurably to make students proficient in such identification.

Many of the smaller museums of the country have now installed or are contemplating the installation of planetaria to augment their educational programs. Not only do they present public demonstra-

tions (usually on week-ends), such as the ones given at Cranbrook previously mentioned, but they have special demonstrations on week-days during the school year for classes of children at all grade levels. In addition, special demonstrations are provided for any interested group on request.

Correlated with such museum planetaria are related activities, such as Junior Astronomy Clubs, Amateur Telescope Makers, Moon-watch Teams, and the like, all of which are fostering a newly awakened interest in astronomy in young and old alike.

As planetaria increase in popularity and numbers, there will be an increasing need for a closer correlation between the planetarium demonstration and the work of the classroom; for more adequate preparation in advance of the planetarium visit; and for intelligent follow-up activities in the classroom subsequent to the planetarium experience. Correlations of this nature will require the full support of school administrators and teachers, as well as cooperation between the schools and the planetaria and between school teachers and planetarium lecturers. Unless this correlation and cooperation is forthcoming, it may certainly be said with truth that "although the planetarium demonstration is nearly always an exciting and memorable experience for school children, it is not always as fruitful an educational experience as it might be."

In conclusion it can be said to the extent that "ingenuity has been exercised in exploring and developing techniques which add greatly to the educational value of the demonstration," planetaria have recognized the dramatic impact of the planetarium demonstration as an aid to education. Through the use of music, sound effects, narration and the effects achieved with accessory devices, the planetarium has allied entertainment with education and successfully attracts and dramatically holds audiences, kindles an interest in science among children and adults and ultimately inspires many young people to pursue careers in science.

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## NEW STEP IN MOHOLE PROJECT PLANNED IN PACIFIC AREA

An area planned for the drilling experiments of the Mohole project were surveyed in August. The survey was a preliminary step toward the drilling of a hole through the earth's crust.

The survey was conducted in a ten-mile-square area off Guadalupe Island. This area has been narrowed down from a much larger area originally surveyed off the Mexican coast.

It was learned here that this Pacific area is now "pretty definitely" planned as the site of the experimental drilling in early 1961.

The experimental drilling was designed to test equipment and theory before the big drilling when the earth will be pierced to its mantle, a layer about three miles below the earth floor in this part of the Pacific Ocean. However, the area surveyed may not necessarily be used for the final drilling of the Mohole.

# Explanation of the Two-Cycle Gasoline Engine

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There are always physics pupils who are interested in the two-cycle gasoline engine. Yet most physics books do not explain the operation of this type of engine. And it is difficult to find a book to which to refer pupils. A large museum may have a working model, such as the Smithsonian Museum in Washington, D. C. has, but most communities do not have a museum available.

Many people already know the operation of the four-cycle gasoline engine. The two-cycle engine is similar to that of the four-cycle one mainly in that both are internal combustion engines. However, two major differences are: (1) the two-cycle engine has ports instead of valves such as the four-cycle engine has and (2) the two-cycle engine has a crankcase which plays a major role in its operation. In the two-cycle engine the four processes of intake, compression, power, and exhaust which go to make up the entire sequence of operation are completed when the piston has made one upstroke and one downstroke while in the four-cycle engine these same processes require the piston to make two upstrokes and two downstrokes. In the four-cycle engine the crankshaft makes two revolutions during the four strokes so that the engine would more properly be called the two-cycle engine instead of four-cycle; in the two-cycle engine the crankshaft makes one revolution during the two strokes so that it would more properly be called the one-cycle engine instead of two-cycle.

The complete sequence of events in the two-cycle engine can best be explained with the aid of diagrams. The two-cycle engine has three ports, the inlet port and exhaust port on one side of the engine and the transfer port on the other. The inlet port, labeled I, is opened when the piston P moves up to such a position that the lower part of the piston is above the port, as shown in Figure 1. The inlet port is thereby opened to the crankcase D located below the piston. As the piston moves up, then, the volume of the space below the piston must necessarily increase so that the pressure within the crankcase decreases. This happens just as would happen in a good many pumps of the cylindrical type. The consequence of a partial vacuum in the crankcase is that the mixture of gasoline vapor and air pushes through the inlet port into the space below the piston. As the piston moves up, it also closes the transfer port T and the exhaust port E. Since the combustion chamber C above the piston is now made air-tight, the mixture of gasoline vapor and air which has

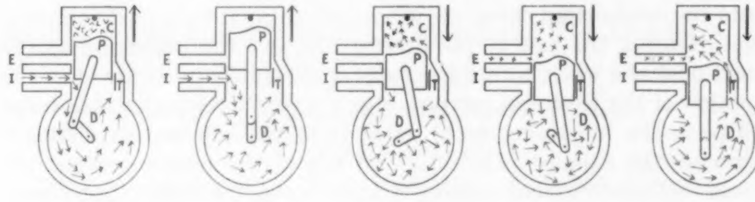


FIG. 1 FIG. 2 FIG. 3 FIG. 4 FIG. 5

P piston; I inlet port; E exhaust port; T transfer port; D crankcase; C combustion chamber; → original fuel introduced; < fuel being compressed in fuel chamber; ← burning fuel mixture; X spent gases

been admitted above the piston previously is compressed. This complete series of events, as explained, consists of: (1) compression of fuel mixture above the piston; (2) exhaust and transfer ports closed; and (3) admission of fuel mixture into the crankcase below the piston.

At the peak of the upstroke, as shown in Figure 2, the fuel mixture in the combustion chamber above the piston is ignited by a spark and as a consequence burns. This series of events consists of: (1) combustion of fuel mixture in the combustion chamber above the piston; and (2) exhaust and transfer ports closed.

The combustion of the fuel mixture within the combustion chamber produces gases which expand and these expanding gases drive the piston downward thereby providing power, as shown in Figure 3. This production of power is the main event for which the engine exists.

As the piston continues to move downward and reaches a position near the end of the downward stroke, the top of the piston moves below the exhaust port and opens it to the combustion chamber, as shown in Figure 4. The residual pressure of the spent gases in the combustion chamber pushes them through this port. Enough of these spent gases escape so that the pressure within the combustion chamber drops to that in the crankcase. The main events are: (1) exhaust of the spent gases from the combustion chamber; (2) inlet port closed; and (3) decrease in pressure of gases in combustion chamber to a value which is less than the pressure of the fuel mixture in the crankcase.

The transfer port T to the right is opened just after the exhaust port is opened, as shown in Figure 5. Since the pressure of the fresh fuel mixture in the crankcase is greater than that of the spent gases that remain in the combustion chamber and since the crankcase is connected to the combustion chamber through the transfer port, the fresh fuel mixture in the crankcase moves through the transfer port

into the combustion chamber, where it is ready for the next compression. During the rest of the downstroke and the beginning of the next upstroke when both the exhaust and transfer ports are open the motion of the fresh gas mixture into the combustion chamber helps to push the remaining spent gases from the combustion chamber through the exhaust. Deflectors on top of the piston prevent the fresh fuel mixture from passing straight across the combustion chamber to the exhaust port while the remainder of the spent gases are being exhausted. This last series of events consists of: (1) a complete clearing of the spent gases from the combusion chamber; (2) the inlet port closed; (3) the transfer and exhaust ports open; and (4) the intake of the fresh fuel mixture into the combustion chamber.

This type of two-cycle engine in which the fuel mixture is ignited is used for outboard motorboat engines. However, the two-cycle engine which operates with the use of air heated by compression has been even more successful. In this two-cycle engine air instead of the fuel mixture is introduced beneath the piston and passes through the transfer port into the combustion chamber above the piston where it is heated by compression. Into the hot air the fuel is sprayed and the burning of the fuel mixture produces the power.

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## BINOCULAR VIEWER, FILTERS MAKE COLOR PICTURES

Two black-and-white slides of the same subject, when taken through red and green filters and viewed with corresponding filters through binoculars, will yield a full-color picture.

Drs. Norman Geschwind and John R. Segal of the Veterans Administration Hospital's department of neurology in Boston report in the journal *Science* that a full-color picture also can be seen if one of the filters is not used in the viewing process. This investigation confirms with binoculars the same effect recently studied by Dr. Edwin H. Land of the Polaroid Corporation. Dr. Land, extending British work of 45 years ago, produced a full-color picture on a screen by projecting two black-and-white slides through suitable filters.

The Boston investigators used crossed polarizing screens to adjust the color balance of the picture, and the viewer's brightness control to adjust the colorful brilliance.

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## RARE EARTHS, NOT RARE, FINDING NUCLEAR JOBS

The rare earths, actually not at all rare in nature, are finding more and more jobs to do in industry, particularly in the field of nuclear ceramics. The demand for the 15 elements of this series will increase in the future.

This was the outlook presented for these ceramic-like materials that have high melting points. This quality appears to make them suitable for such new uses as crucibles in which metals, glass and enamels can be melted, and also for jobs where a material must withstand high nuclear radiation. Rare earths can be used in control rods for nuclear reactors, and also as a radiation shielding ingredient in concrete.

# Recipe for Summer School Science—Take 'Em Outdoors

Harold R. Hungerford
*Principal, West Junior High School, Kankakee, Illinois*

Guisto Patinella
*Science Instructor, West Junior High School, Kankakee, Illinois*

A group of thirteen junior high school students stood on the bank of a midwestern stream and filled the air with a variety of squeals and comments. One boy cautiously but tanaciously held on to a wriggling water snake which he had just captured from a dense thicket of water weeds in a shallow area of the stream. For the first time in their lives, these thirteen students were to realize that not all water snakes are poisonous and that they play an important part in the ecology of a stream or river.

The instructor gives two ninth grade students an assist in seining operations as the class searches for a variety of aquatic life during summer school in Kankakee's summer science program for junior high school students.

The casual observer at this particular point on Exline Creek in Kankakee County, Illinois, would never think that this was a formal science class receiving instruction and credit in science through the summer school program at the Kankakee Public Schools. Perhaps the course was not too formal, but the exhuberance reflected by the students should not have been mistaken for poor conduct, but rather wholesome excitement over biology studied in the field.

A group of seventh, eighth, and ninth graders dressed in swimming suits and sneakers, carrying nets, jars full of formalin, minnow buckets, and a variety of other materials and notebooks, can provide most most any school system with an interesting and satisfying experience

A fish specimen is being examined by class members in the field science course. About seventy-five separate aquatic species were collected and identified during this six week program.

in science education. These thirteen junior high school students were present because of their own interest in science. A dream was finally being realized and an experiment in outdoor science instruction was yielding very satisfying results.

For some time the local science department, in conjunction with the administration, had felt that an enriched and broadened science experience for junior high school students was needed. One excellent possibility for fulfilling this felt need was to take the student into the field where the science experience could really be first hand. Summer school provided the possible vehicle for such a program. It was agreed finally to offer field biology for capable students on an experimental basis. The field problem would be in the area of stream ecology.

It was hoped that this experiment would answer certain basic questions: (1) could students, if motivated, understand and take an interest in stream ecology; (2) could junior high school students learn to use biological keys to identify specimens; (3) what were the possibilities of setting up a museum collection; (4) how well would adolescents adapt themselves to field research; and (5) would an interest in this type of biological community be maintained for an entire summer school session?

The answer to all of these questions proved to be yes. This was the case, even though the procedures involved and some materials used were of high school or even college level.

The area the class selected for study was a portion of a stream flowing through a farm not too far from town. Permission was easily obtained to conduct the study and establish a field site. The students first staked out an area $200' \times 1000'$. The area chosen included both banks, a variety of bottom and current conditions, and most of the aquatic environments to be found in the stream. This provided an opportunity to compare life existing at all levels and in a variety of ecological conditions. Formal class sessions were held to brief students on field techniques, equipment and the use of reference materials. Forms were devised which were used to record pertinent field data relative to specific conditions and collected specimens. In this way a running account could be filed for reference purposes. During the remainder of the course, four-hour excursions to the field were taken three days per week. Students were divided into work groups, some collecting bottom fauna with the use of homemade sieves, screens, and galvanized tubing. Others seined for fish in specific areas, while still other students hunted for reptiles and amphibians. Microscopic samples were studied and an attempt was made to find the initial point on the food chain. Stomach contents of larger specimens were carefully investigated. All physical stream conditions were noted regularly and recorded. The division of labor was shifted from group to group periodically in order that each could have experience in all

A seventh and a ninth grade student examine the underside of rocks for larval specimens during the field science course conducted at Kankakee, Illinois this past summer.

methods of investigation. Alternate days were spent in the laboratory identifying specimens, setting up the museum collection, and interpreting data.

At the conclusion of five weeks of work, the entire collection of about 75 vertebrate and invertebrate species was taken by the class to the University of Illinois where a day was spent with Dr. Phillip Smith of the Illinois Natural History Survey. Here the students verified their identification and were elated to discover that almost all of their identifications had been accurate. Here, too, the students were encouraged to continue this type of research since it was found that very few aquatic specimens had been officially recorded from their county.

As the time for summer school nears, all of the original group of students, as well as many others, have been inquiring as to whether they can enroll in the course this summer, even though it means an outlay of money for tuition. It is planned to attempt a comprehensive survey of the county's streams in order to obtain a better idea of vertebrate aquatic life in the county. This will be a challenge at the junior high school level, but it is expected that the survey can be successfully completed with interested and motivated students.

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## DOCTOR URGES STUDY OF TB AS CANCER TREATMENT

It may be that tuberculosis could be used as a "treatment" for cancer. No one is making definite statements, but Dr. Louis Pelner of Swedish Hospital in Brooklyn, N. Y., believes the possibilities of using artificially induced bacterial infections such as tuberculosis should be studied.

The idea of using one disease to fight another is not new. The fever of malaria has been used to "burn-up" syphilis parasites.

In 1891, Dr. W. B. Coley tried to produce erysipelas, an infection with Streptococcus pyogenes, in ten patients with inoperable cancer. But the procedure was difficult and dangerous so be began to experiment with mixed bacterial toxins.

Patients are still alive who have recovered from inoperable sarcomas after having been treated with a toxin mixture by Dr. Coley, Dr. Pelner asserted in the Journal of the American Geriatrics Society.

In 1916, Dr. W. M. Dabney gave tuberculin to seven cancer patients. One showed "remarkable" improvement, which continued after three months of tuberculin therapy, and another showed "unexpected improvement."

"Unfortunately," Dr. Pelner states, "no further follow-up was undertaken, so the final outcome of this form of therapy is unknown."

Another report in the Journal states that more new cases of pulmonary tuberculosis are now being found in persons past 50 than in any other age group.

# The Dimensions of Rydberg's Constant

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In a number of intermediate chemistry texts[1] a formula is given, or derived, for Rydberg's constant, $R$, in terms of the so-called fundamental constants:[2]

$$R = (2\pi^2 me^4)/ch^3.$$

Here $m$ and $e$ are the mass (or reduced mass) and charge of an electron, $c$ is the velocity of light, and $h$ is Planck's constant. Substitution of the appropriate numerical values, using electrostatic units, leads to a value of $R$ that agrees with one obtained from spectroscopic data, 109,677 $cm^{-1}$. When a student carries out the calculation he is usually very impressed with the success of the Bohr theory which led to the formula; so impressed that only rarely does he notice that his units do not agree. If he does check his units he will find that $R$ is in $statcoul^4sec^4g^{-2}cm^{-7}$—a far cry from what is desired, numerical agreement notwithstanding.

This difficulty arises from the fact that the derivation is made using an incorrect form of Coulomb's Law.[3] The expression $F = k(q_1q_2/r^2)$, or $(1/\epsilon)(q_1q_2/r^2)$, or $(1/4\pi\epsilon)(q_1q_2/r^2)$, is always reduced to only $F = q_1q_2/r^2$. If we are using electrostatic units this is justifiable, with respect to arithmetic, by the fact that in empty space $F$ is 1 dyne when $q_1$ and $q_2$ are each 1 statcoulomb and $r$ is 1 centimeter. In this case $k$, or $\epsilon$, or $4\pi\epsilon$, is equal to 1. But while we can drop the 1 we cannot drop the units it carries, $statcoul^2sec^2g^{-1}cm^{-3}$ (for $\epsilon$).[4]

When the derivation of $R$ is carried out with the dielectric constant left in the statement of Coulomb's Law, it is readily seen that the formula for $R$ contains the term $\epsilon^{-2}$. The units of $R$ are then $(statcoul^4sec^4g^{-2}cm^{-7})(statcoul^2sec^2g^{-1}cm^{-3})^{-2}$, or $cm^{-1}$, as we know they must be.

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[1] For example—

Cartmell and Fowles, *Valency and Molecular Structure*, Butterwort, 1956, p. 15.

Gilreath, *Fundamental Concepts of Inorganic Chemistry*, McGraw Hill, 1958, p. 98.

Kittsley, *Physical Chemistry*, Barnes and Noble (College Outline Series), 1955, p. 153.

Nekrasov, *Kurs Obshchei Khimii* (A Course in General Chemistry), Gos. Nauch-Tekh. Izd-vo. Khim. Lit., 1953, p. 117.

Pauling, *General Chemistry*, 2nd ed. Freeman, 1953, p. 177.

Richtmeyer and Kennard, *Introduction to Modern Physics*, 3rd ed., McGraw Hill, 1942, p. 235.

[2] Alternatively, an expression may be given for the energy of an electron in the $n$th orbit of a hydrogen atom, or for the radius of such an orbit. My criticism of the method of derivation applies to these cases as well.

[3] The only text that has come to my attention that does not make this error is F. W. Sears, *Principles of Physics*, vol. 3, Addison-Wesley, 1945.

[4] The dielectric constant may be considered to be the fundamental unit rather than charge. Then $\epsilon$ has no units other than "dielectric constant" but charge may be resolved into $\epsilon^{1/2}g^{1/2}cm^{3/2}sec^{-1}$.

Although the point may seem trivial my experience has been that it causes difficulty for the student who is careful enough to uncover it; there is a mental block that prevents him from suspecting the presence of an error in a textbook. If the derivation is shown he might catch the missing term in the statement of Coulomb's Law, but if the formula alone is presented it would be an exceptional student that could show the absence of the $\epsilon^{-2}$ term.

In addition, such a treatment represents a denial of the value of dimensional analysis so stressed in most physical science courses and encourages sloppy thinking in its implication that numerical agreement is the sole criterion of the agreement between theory and experiment.

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## WORLD'S LARGEST SOLAR TELESCOPE TO BE BUILT AT KITT PEAK NATIONAL OBSERVATORY

Construction of the world's largest solar telescope was scheduled to begin at Kitt Peak National Observatory in September of this year.

"When completed, this huge instrument will give solar researchers more revealing views of the sun than have ever been possible from earth," said Dr. Alan T. Waterman, Director of the National Science Foundation. "Such observations will increase substantially man's meager knowledge of the star that keeps our planet alive."

Solar images as large as 34 inches in diameter will be formed by the Kitt Peak giant. The Kitt Peak solar telescope will represent a significant astronomical research advantage for this nation. Plans call for completion of the solar facility in approximately two years. It will be available to all qualified scientists for research purposes.

The National Science Foundation has granted $4 million for the Kitt Peak solar program in which the huge sun telescope plays a major role. The instrument will have a focal length of 300 feet and will form images of the sun several times larger and more brilliantly illuminated than are attainable with any other solar telescope. It will be made up of three large reflecting surfaces combined in a system requiring a supporting structure about the size of a 10 story office building.

An 80 inch flat mirror, termed a heliostat, will stand 110 feet above the ground atop Kitt Peak, a 6,875-foot mountain located 40 miles southwest of Tucson on the Papago Indian reservation. Sunlight striking the heliostat will be reflected to a 60 inch parabolic mirror mounted 480 feet away at the bottom of a shaft cored into the mountain. From this point light will be reflected an additional 280 feet to a 48-inch mirror which will direct it into an underground observing room. There the sun's image—almost a yard in diameter—may be photographed or directed to spectroscopes, devices that divide light into its component parts and permit study of each part.

The big instrument will enable researchers to study the sun in much greater detail than is possible with any present solar telescope. In addition to spectral investigations, scientists hope to learn much more about sunspots and solar "flares," phenomena which affect radio and other forms of communication on earth.

# The Rudiments of Probability Theory

Joseph Mayer

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## I. Definition and Measurement

The theory of probability deals with questions such as the following: What is the chance of turning up heads in the flip of a coin? Of drawing a heart from a pack of cards? Of the next child born in Chicago being a girl? Of a certain comet striking the earth on its next visit? Of a 50-year-old man dying during the coming year?

Some probability questions like these are simple to answer; others, much more difficult if not impossible. The simplest problems can readily be demonstrated from unbiased games of chance, out of which in fact intuitive or mathematical probability theory took its rise.*

An important reason for taking illustrations from unbiased games of chance is that here several ideal conditions can usually be assumed to be present. One of these is *equally-likely* happenings. In the coin-tossing example, if the coins are properly balanced either heads or tails are equally-likely events and the chance of the coin falling heads up is one chance in two. With a well-shuffled pack of unaltered cards, there are 52 equally-likely happenings and the chance of drawing a heart is 13 out of 52, there being 13 hearts in a deck.

A second ideal condition present in simple games of chance are *mutually-exclusive* events. In the throw of a die or the toss of a coin, only one side can turn up; the others are all excluded. A third ideal condition is to be able to secure ***an exhaustive or complete set*** of possible events to serve as a total against which favorable chances can be measured, such as the two sides of a coin, the 6 sides of a die, and the 52 cards in an ordinary deck of cards. . . .

The simple examples here given satisfy one's intuition or common sense. They represent basic conditions in the theory of ***a priori*** or mathematical probability, and they provide an easily-understood means of measuring probability.[1]

Before going further, however, we need to have in mind other ideas basic to the theory of probability, most of which are also intuitive.

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* Statistical probability, as found in life-expectancy tables, is based on accumulations of factual records and on the actual frequency with which given situations or events occur.

[1] Chance events are by no means all simple. Nor, when complex, are they all equally-likely and mutually-exclusive. But complex probabilities can often be computed from simple probabilities; and, where complex events can be broken down or factored into simple basic events, the complex probab!lities can as a rule be the more readily ascertained.

*A priori* or mathematical probability has recently been labeled, by the statisticians, the "conceptual counterpart" of *a posteriori* probability, As such, it provides models and tests of various kinds applicable to the actual data of experience.

We have already used the words chance and probability interchangeably, without comment. Among other closely-related concepts are possibilities, events, happenings, occurrences, trials, acts, observations, experiments.

Chance is thought of as pertaining to something that happens fortuitously or at random, i.e. as the result of largely unknown or unknowable forces, outside any recognized order. A possibility is viewed as something conceivable or as existing apart from its actual happening—as a potential out of which actual events may or may not emerge. The possible rain in a dark cloud may or may not fall on a given area. Thus every probable event is first of all a possible event. But not all sets of possible events form a basis for probability—only those out of which an expected event emerges at random or (as we say) by chance.

The ultimate concept in probability theory has come to be regarded as an expected or selected *chance event*, related to a larger and complete set of possible events of a certain kind, of which the expected event forms a part. Thus heads may be viewed as the expected chance event out of two potentially possible events and the card in hearts as one of a group of 13 expected events out of a full set of 52 possible events.[2]

Besides the chance event and the complete and proper set of possible events of which it forms a part, there is a third group of ideas, also basic in probability theory. These are the deliberate *acts* of man involved in setting the stage or providing the opportunity for the expected chance event to happen or fail to happen. Various terms are here employed, such as trial, act, experiment, observation; and the results are noted as successes or failures, favorable or unfavorable outcomes, or merely that the selected event happens or does not happen.

Here *a priori* probability, based on intuition, and *a posteriori* probability, based on experience, find opportunity to check one another. If the chance of heads turning up is intuitively thought of as one chance out of two, this obviously does not mean that, if a coin is tossed twice, the experimenter is bound to get heads at least once.[3] However, if he tosses a coin 1000 times and achieves something like

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[2] While, colloquially speaking, "happening" and "occurrence" are frequently used synonymously with "event," the concept "event" seems to have more of a potential or neutral significance with regard to actual functioning. To speak of the *occurrence* or *happening* of an event, moves it from a potential or possible status to one of action or performance. "Event" may thus be used as both "possible" and "probable," which are its dual roles in the theory of probability.

[3] Each unbiased toss in such a situation is independent of every other, and the chance in each remains the same. The experimenter may turn up heads (or tails) six times in a row, but these completed trials have absolutely no effect on the probability of turning up heads (or tails) on the next trial, unless some acquired trick or bias has entered the tossing process.

500 heads, experience tends to confirm intuitive belief; 500 successes out of 1000 trials is also (on balance) one out of two. But more of intuition and experiment elsewhere. Here we wish to emphasize mainly the three sets of circumstances outlined above which are fundamental in the theory of probability. . . .

We may now take up the basic measurement of single-trial probability. If a doctored coin has heads on both sides, the occurrence of heads on a toss of this coin is not only highly probable. It is absolutely certain. To secure tails with such a coin is not merely slightly probable. It is absolutely impossible. Varying degrees of probability may thus be thought of as lying between these two extremes—the extremes of impossibility and certainty.[4] These extremes may logically be designated as zero (0) and one (1), with actual probabilities ($p$) as common fractions or decimals ($0<p<1$) lying in between. For example, the occurrence of heads in the toss of an unbiased coin has a probability of 1/2 or .5; to draw a card in hearts has a probability of 13/52 or .25.

### Basic Definition of Simple, Single-trial, Unconditional Probability[5]

If out of a total of $(m+n)$ possible events of a certain character (equally-likely, mutually-exclusive, and forming a complete set), there are $m$ favored events, the probability ($p$) that any one of the $m$ events will happen in a single trial is $p=m/(m+n)$ and the probability ($q$) that such an event will fail to happen is $q=n/(m+n)$.[6]

An immediate result of this definition is that $p+q=1$. Since the favored event will either happen or fail to happen, the sum of $p$ and $q$ constitutes a certainty, equal to unity by definition.

Two theorems follow, the first of which stems immediately from the basic definition.

*Basic Theorem I.* (The Addition Theorem, based on "either . . . or"). The probability of ***either*** one ***or*** the other of two selected random events (either $A$ or $B$) occurring in a single trial, is the sum of their separate probabilities, provided $A$ and $B$ are ***mutually exclusive*** and ***non-duplicative***.[7] This "either . . . or" addition theorem may be extended to any number of events or groups of events. Moreover, it does not require that the groups of events ($A$, $B$, $\cdots$) from which

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[4] Whether these extremes should also be viewed as "probabilities" is a question not pertinent to the present article.

[5] For complex or compound probabilities and for repeated trials, additional rules have been developed, as will be indicated later on.

[6] The expression "successful or unsuccessful outcomes" is frequently used for "selected events happening or failing to happen."

[7] For modifications of these Theorems, where duplications or dependent events are involved, see below, p. 559. The inclusion of "or both" is not implied in Theorem I.

the combined probability is derived must necessarily, as groups, have the same initial probabilities.

*Examples:* The fact that $p+q=1$ may be construed as an example of Theorem I; every selected chance event will *either* happen *or* fail to happen. Again, if a coin and a die are tossed together, we have a situation in which the respective probabilities (1/2 and 1/6) are not the same; nevertheless, the combined probability, that (say) *either* heads *or* a six will turn up, is the sum of the separate probabilities, i.e. $1/2+1/6=2/3$. The drawing of a queen of hearts on a single try has a probability of 1/52, as has also the drawing of a jack of hearts or a ten of hearts. To draw any one of the 13 cards in hearts on a single try is again a sum and has a probability of $1/52+1/52 \cdots$ (13 times) or 1/4.

*Basic Theorem II.* (The Product Theorem, based on "both . . . and"). The compound probability that two selected random events *both A and B*) will occur together in a single trial, is the product of their separate probabilities provided $A$ and $B$ are *independent* of each other. This product theorem may also be extended to any number of events or groups of events and, with certain modifications, to dependent or conditional events.[7] In the application of this theorem, the groups of events involved need not, once more, have the same initial probabilities.

*Examples:* The throwing of two sixes with two dice on any one try has a probability $1/6\times1/6=1/36$.[8] The securing of heads with a coin and a six with a die, on a single throw of both together, has a probability of $1/2\times1/6\doteq1/12$. To draw two hearts from two separate packs simultaneously has a probability of $1/4\times1/4=1/16$. This is also true for successive draws, one from each pack, but not for two draws from the same pack unless the first card drawn has been replaced.[9]

*Theorem III* (Combination of Addition and Product). Theorems I and II may be combined in calculating the probability of a composite event. Such a combination is frequently encountered even in relatively simple situations.

*Example:* The securing of a total of six, on any one throw with two dice, is an illustration. There are 5 ways of throwing a total of six with two dice, each of which (by the product theorem) has a probability of 1/36. The total probability (any one of the five ways) thus adds up to 5/36. . . .

The attempt thus far has been to limit the exposition to single trials and to simple probabilities or basic combinations of them, more

[8] A simple listing of possibilities will show that, when two dice are thrown, there are $6\times6$ or 36 total ways in which they may fall.

[9] Without replacement, $p=13/52\cdot12/51=1/17$ instead of the 1/16 above. See, also, p. 563 below.

complex situations and the effect of a number of trials being left for later. Before additional questions are taken up, however, it will be helpful to analyze the bearing of permutations and combinations in more complicated situations. In fact, the rules of permutations and combinations make it possible to determine, by separate computations, the number of successful or favorable events (constituting the numerator) and the total number of possible events (constituting the denominator) of the common fraction ($0<p<1$) measuring the probable happening of a selected chance event or group of events, especially in complex situations. The five in the numerator and the 36 in the denominator, in the preceding example, may be viewed as an application of such rules in a relatively simple problem.

## II. Permutations and Combinations

Despite the simple single-trial illustrations given in the preceding section, events as we ordinarily meet them are neither simple nor limited to single trials. Events are as a rule composite and frequently need to be broken down or disassociated into simpler components. sometimes analogous to the factoring of a composite number, in order to determine how many elemental events of a certain kind or of different kinds are involved in a given problem.

Composite events may require resolution into primary elements, called permutations, or into intermediary elements, called combinations. By definition, permutations of a given composite event are as a rule more numerous than its combinations.

*Examples:* (1) The number 625 may be resolved into *two* intermediary factors ($25\times25$) or into *four* primary factors ($5\times5\times5\times5$). (2) The letters $a$, $b$, $c$, taken two at a time form three combinations ($ab$, $ac$, $bc$), the order in which the letters in the three sub-sets occur being disregarded. The permutations here involved are six ($ab$, $ba$, $ac$, $ca$, $bc$, $cb$), the order of the letters in the three combinations being taken into account also.

The composite events thus encountered are frequently spoken of as groups of "objects" or "things" and their combinations or permutations as "ways" in which their elements may be arranged. In such groupings or arrangements, three considerations are important: The kinds of elements included in a given group; the number of elements of each kind involved (if more than one kind); and the ways in which the elements of a group or sub-group are arranged. Permutations take all three of these considerations into account; combinations, the first two only. Put into the form of definitions, we have:

*Permutations* (P): The total of every variety of ways and orders in which a selected set of events or parts of a given set of events may be arranged.

*Combinations* (C): The total number of groups in all or parts of a set of events, without regard to the arrangement of the elements in the groups or sub-groups.

In dealing with permutations and combinations, Theorems I and II (outlined in the preceding section) are just as applicable to *events* as they are to *probabilities*. In the language of events, these theorems may be rephrased as follows:

*Theorem I* (addition): If an event $A$ can occur (or "object" be arranged or "act" be performed) in $m$ different ways, and an event $B$ can occur in $r$ other ways, either $A$ or $B$ can occur in $m+r$ ways.[10]

*Example:* If there are four ways of going from Oxford to Cincinnati and three of going from Oxford to Richmond, Ind., there are seven ways of going from Oxford *either* to Cincinnati *or* to Richmond.

*Theorem II* (product): (a) If an event $A$ can occur (or "object" be arranged or "act" be performed) in $m$ different ways, and an event $B$ can occur, either simultaneously or in succession, in $r$ other ways, both $A$ and $B$ can occur together in $mr$ ways, provided $A$ and $B$ are unconditionally independent of each other. (b) If the relationship between $A$ and $B$ is dependent or conditional, their occurrence must be in succession, the order of succession producing a modified product ($mr'$ or $rm'$).[11]

*Examples:* (1) In the preceding example, a journey from Cincinnati to Richmond through Oxford could be undertaken in $4\times3$ or 12 ways, since the routes from Cincinnati to Oxford are independent of the routes from Oxford to Richmond. (2) How many permutations or choices are there between six different letters taken three at a time with no letter used more than once? Here the first place or position in the three-letter sequence (or any place or position of the three) can be chosen in 6 different ways without condition or qualification, but when that place is filled there are only 5 letters left for a choice of the second place, and only 4 in choosing the third. Thus the three places or positions can be filled in $6\times5\times4$ or 120 different ways. (However, if a letter chosen may be counted again, the choices become independent of one another and the number of permutations are $6\times6\times6$ or 216). . . .

The conditional or dependent relationships covered in Theorem II (b) form the basis for some important mathematical formulations. Generalized, the calculation in the second of the foregoing examples $[6\times(6-1)\times(6-2)]$ becomes:

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[10] In this restatement, the phrase "$r$ other ways" implies that the ways are nonduplicative and mutually exclusive.

[11] For confusion between "successive" and "simultaneous," see below, p. 563.

(1) $$P_r^m = (m-0)(m-1) \cdots (m-r+1)$$[12]

This is the preferable formula to use for small values of $m$ and $r$. For large values a table of factorials may be employed, with formula (1) converted into the following form by multiplying both numerator and denominator by the factorial $(m-r)!$

(2) $$P_r^m = m!/(m-r)!$$

Three special cases are important here: If $r=0$, formula (2) becomes $m!/m!=1$; if $r=1$, we have $m(m-1)!/(m-1)!=m$; if $r=m$, we have [from (1)], $P_m^m = m!$, [from (2)] $P_m^m = m!/0!$, from which two versions, to maintain consistency, we define $0!=1!=1$. . . .

Formulas for combinations are readily derived from permutation formulas (1) and (2), by utilizing the fact that each combination is capable of $r!$ permutations within itself and that these are not counted in the combinations. This may be illustrated by the second example at the beginning of this section. There three letters taken two at a time gave 6 permutations and 3 combinations. There, $r!=1\times 2$ and $P_2^3 = m(m-1) = 6$. Dividing $P_r^m$ by $r!$ gives 6/2 or 3 combinations, as before.

Generalized:

(3) $$C_r^m = P_r^m/r! = m(m-1) \cdots (m-1+r)/r!$$

(4) $$C_r^m = m!/r!(m-r)!; \quad \text{also,} \quad C_{(m-r)}^m = m!/r!(m-r)!,$$

from which it follows that $C_r^m = C_{(m-r)}^m$, an important identity for simplifying certain calculations.

These formulas assume that the elements contained in $m$ are all different. However, if in $m$, $s_1$ elements are the same (likewise $s_2$, $s^3$, . . . ), then the foregoing formulas will need to be divided through by $s_1!$, $s_2!$, $s_3!$ . . . in order to take account of these groups of similar elements, or, rather, to take them out of account. Thus:

(5) $$P_m^m = P_m^{s_1 s_2 s_3} = m!/s_1!s_2!s_3!$$

(Note that in this formula $m$ is taken $m$ at a time and that $s_1+s_2+s_3=m$)

*Example:* The letters in the word Illinois can be arranged in how many different ways, omitting duplications? There are three repetitions of one letter and two of a second letter. Therefore,

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[12] In the example given, $m=6$, the total number of elements involved, and $r=3$, the number of the $m$ elements used at a given time in carrying out the operation. The symbol $P_r^m$ is one conventional way of stating such an operation.

$$P_8 8(3, 2, 1, 1, 1) = 8!/3!2! = 40{,}320/12 = 3{,}360 \text{ ways.}$$

If the duplications are counted, the total number of different ways is raised to $P_8{}^8 = 40{,}320$. If the number of combinations of these letters to get the word Illinois is sought, the answer is one, i.e. $C_8{}^8 = 8!/8! = 1$. · · ·

The proper application (with some simple extensions) of these rudimentary formulas, theorems, and definitions suffice for the solution of most problems in elementary probability. To go farther than this is not the purpose in the present article, except in the concluding section to indicate the bearing of these general rules upon one of the most fundamental theorems of both mathematical probability and statistics, viz. the binomial expansion.

Three illustrations of the combined use of the foregoing formulas and theorems are added here:

*Example 1.* Seven balls, numbered one through seven respectively, are placed in a hat and shaken. Four are drawn *in succession* without replacement. Find the probability that the numbers on the first and fourth draws are odd.

This problem can be solved by a direct application of Theorems II (b) and I. The rules of permutations apply because the *order* of the drawings is involved. The total number of possible arrangements or outcomes (the denominator of the probability fraction) is $P_4{}^7$ or 840. The number of ways to get odd-numbered draws first and last (Theorem IIb) is $4 \cdot 5 \cdot 4 \cdot 3$ or 240. Thus 240/840 or 2/7 is the probability in question.

*Example 2.* A jar contains 5 red and 6 green marbles. After shaking, four are drawn without replacement, the order being disregarded. What is the chance that (a) two marbles will be red and two green; (b) that all four drawn will be of the same color?

The denominator of the probability fraction is the same for both these problems, viz. $C_4{}^{11}$ or 330 possible combinations. For (a), the numerator is secured through $C_2{}^5 \cdot C_2{}^6$ or 150 successful combinations of drawing *both* 2 red marbles *and* 2 green ones. Thus the probability is 150/330 or 5/11. For (b), the numerator becomes $C_4{}^5 + C_4{}^6$ or 20 successful combinations of drawing *either* 4 red marbles *or* 4 green marbles. Hence the probability here is 20/330 or 2/33.

*Example 3.* Six cards are drawn without replacement from a deck of 52 cards, the order of the draws being disregarded. What is the chance that of the six cards drawn 3 will be black and 3 red. The favorable draws (numerator) are $C_3{}^{26} \cdot C_3{}^{26}$ and the total possible combinations (denominator) are $C_6{}^{52}$. Thus the probability is

$$(C_3^{26})^2 \div C_6^{52} = 13,000/39,151 \quad \text{or} \quad .332.$$

## III. Some Semantic Considerations

Before turning to binomial expansions, several difficulties and distinctions, not always clearly understood, should be noted.

1. The concepts of randomness or chance, equally-likely happenings, mutually-exclusive events, and complete sets of assignable possibilities require further consideration as one turns from simple single-trial situations to multiple trials and complex or compound events.[13]

Pure chance and equal likelihood are parts of the same intuitive probability idea. However, in positing a complete set of possible elemental events, each of which (so far as we know) stands an equal chance of occurring, we are confronted with only the simplest and most primative of probability situations. Equal likelihood applies only to the elements of a specific primitive set. The equal likelihood of a happening in one set (e.g. a six-sided die), as has been seen, may be quite different from an equal-likely happening in another set (e.g. 52 playing cards),—one in six as against one in 52. In complex situations it is important therefore to hold in mind that complete randomness or chance requires equal likelihood within primitive sets but not in comparisons between sets or in combinations of them.

Partial randomness or deviations from pure chance are also encompassed in probability theory, as in the case of a loaded die, provided the likelihoods of each elemental occurrence can be independently assessed, through experiment for example. After basic probabilities have been determined, by whatever means, the addition and multiplication theorems apply without further recourse to the equal-likelihood criterion. In fact, attempts to use this criterion, in many advanced stages of probability analysis, would be beside the issue or would lead to confusion or error.

With respect to mutual exclusiveness the situation is different. Not only is this criterion important with respect to the elements of primitive sets, but it also continues to remain significant with complex or compound events and with relations between sets and subsets.

Mutual exclusiveness restricted to single trials and primitive sets covers several inter-related concepts: First, the elements or subsets involved are all separate or disjoint; second, each element or subset differs from all the others, i.e. there are no duplications; third, only

[13] Questions regarding infinity, continuous variation, "insufficient" or "cogent" reasoning as to equal likelihood and chance events, and similar questions are not taken up in this article.

one of the elemental events or subsets can happen in a given trial or experiment, i.e. all the other possibilities are excluded.[14]

This is not to imply that duplications are rare. Where they are known to exist, they can as a rule be readily taken care of (as in the probability of drawing a heart from a pack of cards;—since there are thirteen duplicative hearts, the probability becomes thirteen out of 52). A frequent difficulty, however, is to spot the duplications, especially where they are not obvious on the face of the problem.

*Example:* Two cards are drawn in a single trial, simultaneously, one from each of two identical packs; what is the probability that either one or the other card is an ace of hearts? Since there are two aces of hearts in the packs and the appearance of both together is not part of the problem, the possibility of a double or duplicative appearance must be taken into account. The answer is thus $1/52+1/52-1/52\cdot 52=103/2704$, making use of both the addition and product theorems.[15]

2. Mutual exclusiveness pertains to non-duplicative *elements* or *subsets* within a set of events, whether the set is simple or compound, not to multiple *trials* which by their very nature are duplicative. The latter pertain to repeated duplicative experiments with a given set of events taken as a whole. Thus the appropriate designations here are independent versus dependent trials and unconditional versus conditional situations. It is dependent or conditional trials which lead to the important permutation and combination formulas outlined in the preceding section.

Trials (experiments, observations) are independent of one another if the outcome of any one of them has no effect on the outcome of the others. Whether such independent trials are performed simultaneously or in succession is immaterial. This makes it possible to view certain problems in two different ways, occasionally confused.

*Example:* If twelve dice are involved in a problem, throwing them simultaneously would naturally be viewed as one trial of a compound event composed of twelve mutually-exclusive sub-events, while throwing them in succession would be regarded as twelve trials of a six-element event, the faces of each die being the mutually-exclusive elements. In probability analysis, both views come to the same thing where mutual exclusiveness and independence are involved.[16]

It should be noted, however, that the term "simultaneous" is

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[14] In the Weldon experiment (described on p. 564, below), conditions are imposed on the fall of 12 dice which divide them into two sets and allow for duplicative successes and failures. Underneath, however, there are still 12 separate dice and six faces per die, each (in proper primitive relationship) being nonduplicative and mutually-exclusive.

[15] Here the difference in the probabilities is very small, .0381 taking the duplication into account as against .0385 if the duplication is overlooked.

[16] Though either view or approach gives the same result, one approach may provide a simpler or quicker solution than the other, as in the de Méré problem, regarding which see pp. 564–565, below, including footnote.

sometimes improperly used, as when it is said that two cards are drawn from the same pack "simultaneously." Here it is clearer to say that the cards are drawn in succession, either without replacement or with replacement: The one situation involves a ***dependence*** of the second draw on the first; the other situation, ***independent*** draws. Two cards may, of course, be drawn simultaneously from different packs or there may be two or more tosses of coins or dice simultaneously, which events are always independent or mutually exclusive of one another.

In making use of the binomial expansion in probability theory, we deal always with a series of two or more mutually-exclusive elements[17] in a compound *event* and one or more independent ***trials*** or observations or experiments with respect to such an event.

3. In connection with binomial probability distributions, therefore, it is desirable to use different symbols to represent the mutually-exclusive elements or subsets ($n$) in the compound event and the independent experiments or trials ($N$) pertaining to these elements or subsets. Thus we may speak of $N$ throws or trials of $n$ dice tossed simultaneously, to represent, say, 100 independent trials of 12 mutually-exclusive elements in a compound event. By arbitrarily restricting the use of $n$ and $N$ in this manner, the one symbol is always the exponent and the other the coefficient in a binomial expansion, i.e. $N(p+q)^n$,—in the present illustration, $100(p+q)^{12}$. A necessary part of the effective use of the binomial theorem in probability analysis is the inclusion of the probability of failure ($q$) as well as the probability of success ($p$).[18]

4. At least, at most, and exactly are three expressions frequently encountered in binomial probability. To avoid confusion, "at least" is converted into "or more," and "at most" into "or less." "At least three successes" means "three successes or more than three"; and "at most three successes" means "three successes or less than three." By "exactly three successes" is obviously meant "just three, no more or less."

## IV. Binomial Expansions and Frequency Distributions

Certain problems may be solved by either the Addition and Product Theorems or by a Binomial Expansion, as illustrated below:

*Examples:* If the probability of your winning a game of golf on a certain course is 1/3 whenever you play, what is the chance of your winning: (a) just one game in two matches; (b) winning both games; (c) winning at least one game; (d) losing both games?

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[17] Divided into successes ($p$) and failures ($q$), as below.

[18] As used in binomial expansions, the probabilities $p$ and $q$ are of course always complementary, i.e. their sum is equal to 1. Since $p=1-q$, it is sometimes simpler to calculate $q$ directly and then secure $p$ from $1-q$, especially in "at least one success" situations, e.g. in de Méré's problem, pp. 564–565, below.

The conditions of (a) will be satisfied either if you win the first game and lose the second or if you lose the first game and win the second—by either $p_1q_2$ or $p_2q_1$, i.e. $(1/3.2/3+2/3\cdot 1/3)=4/9$. The chance (b) of winning both games is $1/3\cdot 1/3=1/9$. The chance (c) of winning at least one game (either one game or both) is the sum of (a) and (b) or $4/9+1/9=5/9$. The chance (d) of losing both the first game and the second is $2/3\cdot 2/3=4/9$. The chance of winning both games (1/9) or of winning one game and losing the other (4/9) or of losing both games (4/9) is a certainty, i.e. $(1/9+4/9+4/9)=1$.

This problem also illustrates a simple binomial expansion with $N$ equal to 1 and $n$ equal to 2, i.e. $1(p+q)^2$ or $(1/3+2/3)^2$, which, of course, equals $1/9+4/9+4/9$, as above, and gives the two-game probabilities of exactly two successes, one success, and no successes, respectively.

If, under the same conditions, 90 two-game sets are played, the frequency distribution would be $90(1/3+2/3)^2=10+40+40$, representing not the separate probabilities but the probable frequencies of winning 2 games, one game, and no games.

The oft-quoted Weldon experiment, of throwing 12 dice simultaneously 4096 times, serves as an illustration of comparing actual with theoretical frequencies. On each throw of the 12 dice, where spots 4, 5, and 6 turned up they were counted as successes; where 1, 2, and 3, turned up they were counted as failures. In terms of a binomial expansion, we here have $4096(1/2+1/2)^{12}=1+12+66+220+495+792+924+792+495+220+66+12+1$. These are the theoretical frequencies for 12, 11, 10, etc. . . . down to no successes, over against which Weldon actually secured frequencies, for the same range of successes, of 0, 7, 60, 198, 430, 731, 948, 847, 536, 257, 71, 11, 0, respectively. The possibility of thus comparing theoretical or normal with actual frequencies has been of high importance in the development of statistical analysis.

Several phases of this development may be briefly outlined as illustrative, in concluding the present survey.

(1) Such binomial identities as the following have served as a basis for elaborating many others of importance in further probabilistic analysis: $(p+q)^n=1$; $N(p+q)^n=N$; $q^n=(1-p)^n$. One of these $[q^n=(1-p)^n]$ is of particular significance historically, since a portion of it, i.e. $1-q^n$, is equal to all the $p$-terms and thus represents "at least one success" (one or more) with respect to the binomial expansion $(p+q)^n=1$. These facts were used by Pascal in clarifying a question put to him by the gentleman gambler de Méré in 1654. More importantly, the ensuing discussion is said to have marked the beginnings of modern probability theory.

De Méré's wide experience with games of dice led him to conclude that he came out slightly better than even ($p>.5$), when taking a 50–50 chance of throwing at least one six in four games with one fair die; but that he came out slightly less than even ($p<.5$) on a 50–50 chance of throwing at least one double-six in 24 games with two fair dice. This seeming discrepancy he could not understand, for he reasoned that the chance of a favorable result, i.e. of throwing a six out of six possibilities (one die), should be the same as throwing a double-six out of 36 possibilities (2 dice), if the number of games played were in the same (one to six) ratio or, in this case, 4 games to 24.

What de Méré did not realize was that the two probabilistic situations could not be compared as ratios but had to be figured by way of the summations and powers involved in the binomial expansions, $(p_1+q_1)^4$ and $(p_2+q_2)^{24}$, one minus the last term of each ($1-q_1^4$ and $1-q_2^{24}$) representing "one success or more" (at least one) in each series of games.[19] Substituting the actual values for $q_1$ and $q_2$ we have $p_4=1-(5/6)^4$ and $p_{24}=1-(25/36)^{24}$, i.e. $p_4=.52$ and $p_{24}=.49$. These results are what Pascal worked out for de Méré. They fully confirmed de Méré's practical experience, though not his reasoning.

(2) Combinations formula (4), i.e. $C_r^n=n!/r!(n-r)!$, finds important use in determining the coefficients of the general binomial expansion, which (for probability and other purposes) may be written in the following form:

$$(p+q)^n=C_0^np^nq^0+C_1^np^{n-1}q^1+C_2^np^{n-2}q^2+\cdots[C_x^np^{n-x}q^x]+\cdots$$
$$+[C_{n-x}^np^xq^{n-x}]+\cdots+C_{n-1}^np^1q^{n-1}+C_n^np^0q^n.$$

In this form the binomial expansion throws interesting side-lights on well-known algebraic characteristics: The number of terms is $n+1$; the sum of the exponents of $p$ and $q$ is always $n$; the exponent of $p$ begins with $n$ and decreases in unitary stages to zero, while the exponent of $q$ begins with zero and increases to $n$; the $x$ in the coefficient $C_x^n$ also begins with zero and increases to $n$; the values of the coefficients start with one and increase to a maximum in the middle term or terms after which they decrease in reverse order to one; the coefficients of the end terms, and of the terms equidistant from them, are equal. These characteristics are combined in the compact binomial forms

[19] Note the illustration of the two ways (mentioned on page 563 of vizualizing problems of this nature. Here the $N$ may be regarded as 1 and the questions reduced to single-trial binomial situations, i.e. to 4 dice thrown once $(p_1+q_1)^4$ and 48 dice thrown once $(p_1+q_1)^{48}$. The latter comes to the same thing (with respect to $q$) as 24 "double dice" thrown once $[(p_1+q_1)^2]^{24}$. It is, of course, to be understood that all the elemental events involved in these problems are completely independent and mutually-exclusive of one another.

$$(p+q)n = {}^n\sum_{x=0} C_x{}^n p^{n-x} q^x = {}^n\sum_{x=0} C^{n-x} p^x q^{n-x}.$$

(3) The terms in brackets, in the fuller binomial expansion shown above, are obviously replicas of the compact forms just indicated. They are the well-known $(x+1)$th terms in the algebraic theorem, the first counting from the left end and the second from the right end of the expansion.[20]

Either one of these terms may be used as the general term in the binomial theorem. As $C_x{}^n p^x{}_q{}^{n-x}$, it represents the probability of *exactly* $x$ successes out of $n$ independent and mutually-exclusive events in a trial, where $p$ is the probability of success in a single event.[21] This term together with all that precede it in a given expansion, represent the probability of *at least* $x$ successes ($x$ or more); the same term together with all that follow represent the probability of *at most* $x$ successes ($x$ or less). . . .

Regarding discussions and correspondence between de Méré and Pascal and between the latter and Fermat and other noted scholars of their day, the reader is referred to an interesting account by Oystein Ore, entitled "Pascal and the Invention of Probability Theory," published recently in *The American Mathematical Monthly*.[22]

[20] Since $C_x{}^n = C_{n-x}{}^n$, from combinations formula (4), these identities may obviously be used interchangeably.
[21] The alternate form, $C_{n-x}{}^n p^{n-x} q^x$, represents *exactly* $x$ failures.
[22] May, 1960, pp. 409–419.

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## MILK STILL BEST FOOD IN SPITE OF STRONTIUM-90

Milk is still a most satisfactory protective food, regardless of concern about its strontium-90 content. In fact, a recent issue of *Nutrition Reviews*, reports that milk actually protects against strontium-90 accumulation in the bone.

"The reports released by the U. S. Public Health Service of gradually increasing levels of strontium-90 in milk have led to increasing concern on the part of both professional and lay persons," the magazine says.

But a survey by Dr. Bruce L. Larson, assistant professor of biological chemistry, department of dairy science, University of Illinois, gives strong support to the claim that milk is still man's most satisfactory food, not only in terms of nutrition but in preventing strontium-90 accumulation in the bones of our population.

The survey is reported in the *Journal of Dairy Science* in which Dr. Larson says evidence now indicates that people in the "primary milk-consuming areas" are getting relatively lower levels of strontium-90 in their bones than people in the primary "plant-consuming areas." This is attributed to the high calcium level in milk.

Dr. Larson says "it may be just as wise to increase milk consumption as to try to remove the strontium-90 from the milk."

## Highlights of the 1960 Convention

**Lawrence A. Conrey**

***Vice-President, C.A.S.M.T., University of Michigan, Ann Arbor, Michigan***

The 60th Convention of the Central Association of Science and Mathematics Teachers will be held at the Statler Hilton Hotel in Detroit, Michigan on November 24–26, 1960. The theme of the meeting is, "Challenging Science and Mathematics Students Toward the Horizons of Knowledge."

In keeping with this theme, the opening session on Thursday evening, November 24, considers, "The Challenge of Challenging Students." Initial discussion by a panel will be extended to include those in attendence.

This initial challenge will be extended, in the subsequent two days, to many other challenges and horizons of knowledge.

### Horizons of Knowledge:

Our understanding of how we learn to interpret abstractions, the discovery of organized systems in the tiniest of particles, and pushing our explorations ever outward into the vast systems of outer space are stimulating examples of the ever expanding frontiers of knowledge. These "horizons" will be explored, in our general sessions with the following outstanding speakers:

"Horizons of Knowledge of the Atom"—Friday morning, November 25.

Dr. Leonard Roellig, Department of Physics, Wayne State University, Detroit, Michigan.

"Horizons of Knowledge of the Psychology of Mathematics—Elementary and Secondary"—Friday morning, November 25.

Dr. Henry Van Engen, Department of Mathematics, University of Wisconsin, Madison, Wisconsin.

"Horizons of Knowledge of Space"—Friday evening, November 25

Dr. F. D. Drake, National Radio Astronomy Observatory, Green Bank, West Virginia. Director, Project "Ozma."

"Horizons of Knowledge of the Cell"—Saturday morning, November 26.

Dr. Wayne E. Magee, Research Division, The Upjohn Company, Kalamazoo, Michigan.

In addition, a "horizon" of a different nature will be explained at the annual luncheon on Saturday, November 26. This is the horizon of a device—the teaching machine. Since these devices have received considerable mention in the public press and professional journals, we feel fortunate to have a national authority to explain them:

"Teaching Machines on the Horizon? What Are the Implications for Science and Mathematics Teaching?"

Dr. James Holland, Psychological Laboratory, Harvard University, Cambridge, Massachusetts.

CHALLENGES

All of Friday afternoon and the Section Meetings on Saturday morning will be devoted to various forms of challenge and at various levels. To identify those portions of the program of particular interest to you examine the following categories:

1. *Of particular interest to the elementary-school teacher:*

We are happy to announce that we have an expanded program for elementary teachers this year. The opportunities are:

*In elementary mathematics:*

"Implications of Modern Teaching Trends in Junior-High and High-School Mathematics for the Teaching of Arithmetic in Elementary Grades."

Dr. Bernard Gundlach, Department of Mathematics, Bowling Green State University, Bowling Green, Ohio. Director, Greater Cleveland Mathematics Project.

"Geometry in the Elementary School—A Portion of the School Mathematics Study Group Project."

Lenore John, University of Chicago Laboratory School, Chicago, Illinois—Member, 1960 Summer Writing Project—S.M.S.G.

"Important Concepts in Elementary Mathematics."

Dr. Charlotte Junge, Wayne State University, Detroit, Michigan

*In elementary science:*

Eight different science content sessions will be provided. In each, the resource person will identify concepts and use simple materials to illustrate these concepts. The resource people have had experience assisting many groups of elementary teachers. The content areas involved are; Air and Air Pressure, Astronomy and Space Travel, Geology, Plants, Animals, Magnetism and Electricity, Light, and Weather. Since there will be two groups of four simultaneous sessions each, each elementary teacher will have the opportunity to attend two sessions, selected at the time of registration.

Also, on Saturday morning:

"Sensible Curriculum Approaches to Elementary Science."

Jacqueline Mallinson, Assistant Editor, SCHOOL SCIENCE AND MATHEMATICS and Elementary Science Consultant.

2. *Of particular interest to secondary-school mathematics teachers:*

a. "The Changing Mathematics Curriculum as a Vehicle for Challenge."
Dr. Phillip S. Jones, Department of Mathematics, The University of Michigan—President, National Council of Teachers of Mathematics.

b. "Challenges and Guides to the Individual Teacher Raised by the Various Study Group Projects."
Dr. Cecil B. Read, Mathematics Department, University of Wichita, Wichita, Kansas.

c. Discussion by a panel and the group in attendance of the topic: "What are some sensible content changes in high-school mathematics we should make now?"

3. *Of particular interest to secondary-school teachers of science:*

a. "The Changing Science Curriculum as a Vehicle for Challenge."
Dr. Vaden Miles, Department of Physics, Wayne State University, Detroit, Michigan—Past-President, National Association for Research in Science Teaching.

b. On Friday afternoon, in separate sections, consideration will be given to the writing projects in biology, chemistry and physics: "PSSC or not, what are some content adjustments on the horizon?"
Panel of teachers.
"The Chemical Bond Approach to Introductory Chemistry."
Dr. Laurence E. Strong, Department of Chemistry, Earlham College, Richmond, Indiana. Director, Chemical Bond Approach Project.
"The Chemical Education Materials Committee Approach to Introductory Chemistry."
Mr. Robert Silber, Education Secretary, American Chemical Society, Washington, D. C.—Member, Writing Committee, CEMC Project.
"The Work of the Biological Science Curriculum Study Which May Lead to Content Adjustment on the Horizon."
Phillip Fordyce, biology teacher, Oak Park-River Forest High School, Oak Park, Illinois—Member, Writing Committee, BSCS Project.

c. On Friday afternoon, general science teachers will have an opportunity to consider the impact of change in the high-school courses on the content of general science:
"A New Prestige and Importance for General Science."
David Schulert, Director of Science, Lansing Public Schools, Lansing, Michigan.

d. On Saturday morning, in some section meetings, a series of panels concerned with the role of the individual teacher in revising the content of science courses will open the discussion for the entire group. Separate panels will be provided in general science, biology, and chemistry under the title, "What are some sensible content changes we should make now?"

Make plans now to attend the Detroit Convention. We sincerely believe that the experience will broaden your "horizons."

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## PROPOSED CHANGES IN CASMT BY-LAWS

The following changes in the By-Laws were proposed by the Board of Directors at the Spring Board Meeting in May. Action will be taken on these at the annual meeting in November.

ARTICLE III. OFFICERS
SECTION IV. POWER AND DUTIES OF OFFICERS:
(b) VICE-PRESIDENT:

| *Present* | *Proposed* |
|---|---|
| He shall act for the President in the latter's absence. He shall also serve as a member of the Executive Committee. | He shall act for the President in the latter's absence. He shall serve as a member of the Executive Committee and shall act as Chairman of the Program Committee. |

ARTICLE IV. BOARD OF DIRECTORS
SECTION IV. ELECTION, TENURE OF OFFICE, AND COMPENSATION:

| *Present* | *Proposed* |
|---|---|
| . . . Those candidates declared elected to membership of the Board of Directors shall be the required number of nominees receiving the largest number of votes cast in the annual election as described in Section III (b) of Article III. . . . | The four nominees receiving the largest number of votes cast in the annual election as described in Section III (b) of Article III shall be declared elected to the Board of Directors. In case of a tie the decision shall be made by lot under the supervision of the Chairman of the Nominating Committee and the Secretary of the Association. . . . |
| . . . Vacancies in the Board of Directors or list of officers shall be filled by the Board of Directors at any meeting thereof. A director so chosen shall serve until the next annual business meeting when a successor shall be elected to fill the unexpired term. | . . . Vacancies in the Board of Directors or list of officers shall be filled by the Board of Directors. A director so chosen shall serve for the remainder of the unexpired term, when nomination and election shall take place as provided above. |
| Whenever directors are elected, whether at the expiration of a term or to fill vacancies, a certificate under the seal of the Association giving the names of those elected and the term of their office shall be recorded by the Treasurer and Business Manager in the office of the recorder of deeds where the certificate of organization is recorded. | The entire paragraph is deleted. |

# Problem Department


## Conducted by Margaret F. Willerding

***San Diego State College, San Diego, Calif.***


*This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.*

*All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problem should be well done in India ink. Problems and solutions will be credited to their authors. Each solution or proposed problem sent the Editor* should have the author's name introducing the problem or solution *as on the following pages.*

*The editor of the Department desires to serve her readers by making it interesting and helpful to them. Address suggestions and problems to Margaret F. Willerding, San Diego State College, San Diego, Calif.*

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## SOLUTIONS AND PROBLEMS

**Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions.**

**1. Solutions should be in typed form, double spaced.**

**2. Drawings in India ink should be on a separate page from the solution.**

**3. Give the solution to the problem which you propose if you have one and also the source and any known references to it.**

**4. In general when several solutions are correct, the one submitted in the best form will be used.**

## LATE SOLUTIONS

**2700, 2701, 2702.** *G. P. Speck, Virginia, Minn.*

**2701, 2705, 2708, 2712.** *J. Byers King, Denton, Md.*

**2707.** *Mother Alphonsa Kohne, Arcadia, Mo.*

**2708.** *Mary K. Tulock, Hartford, Conn.*

**2708.** *Felix John, Philadelphia, Pa.*

**2711.** *C. N. Mills, Sioux Falls, S. D.*

**2713, 2714, 2716.** *C. W. Trigg, Los Angeles, Calif.*

**2716.** *Brother B. Virgil, New Orleans, La.*

**2698.** *Proposed by John Satterly, University of Toronto.*

The bisectors of angles $A$, $B$, and $C$ of triangle $ABC$ cut the opposite sides at $D$, $E$, and $F$ respectively. They also intersect one another at the in-center $I$. Draw $DE$ cutting $CF$ in $X$. Show that

$$\text{(a)}\ \frac{CX}{CI}=\frac{a+b+c}{a+b+2c} \quad \text{and} \quad \text{(b)}\ \frac{CX}{XF}=\frac{a+b}{2c}.$$

*Solution by D. Moody Bailey, Princeton, W. Va.*

Let $a$, $b$, and $c$ represent the sides opposite the vertices $A$, $B$, and $C$ of triangle $ABC$. We first obtain two values needed later in the solution of the problem. From elementary geometry we have

$$\frac{AE}{EC}=\frac{c}{a} \quad \text{and} \quad \frac{AF}{FB}=\frac{b}{a}.$$

These may be re-written as

$$\frac{AE}{b-AE}=\frac{c}{a} \quad \text{and} \quad \frac{c-FB}{FB}=\frac{b}{a}\cdot$$

Solving we obtain

$$(1)\ AE=\frac{bc}{a+c} \quad \text{and} \qquad (2)\ FB=\frac{ac}{a+b}\cdot$$

Consider triangle $BCE$ with transversal $AID$. The theorem of Menelaus gives

$$\frac{BD}{DC}\cdot\frac{CA}{AE}\cdot\frac{EI}{IB}=-1 \quad \text{or} \quad \frac{BI}{IE}=\frac{BD}{DC}\cdot\frac{AC}{AE}\cdot$$

Let us replace

$$\frac{BD}{DC} \quad \text{by} \quad \frac{c}{b},$$

$AC$ by $b$, and $AE$ by (1) obtaining

$$\frac{BI}{IE}=\frac{a+c}{b}\cdot$$

Adding unity to both members of this equation gives

$$\frac{BI}{IE}+1=\frac{a+c}{b}+1, \quad \text{or} \quad \frac{BI+IE}{IE}=\frac{a+b+c}{b}\cdot$$

This is equivalent to

$$(3) \qquad \frac{BE}{IE}=\frac{a+b+c}{b}\cdot$$

Let us now turn to triangle $BCI$ with transversal $EXD$ and again use the theorem of Menelaus. We have

$$\frac{BD}{DC}\cdot\frac{CX}{XI}\cdot\frac{IE}{EB}=-1 \quad \text{or} \quad \frac{XI}{CX}=\frac{BD}{DC}\cdot\frac{IE}{BE}\cdot$$

Substituting

$$\frac{c}{b} \quad \text{for} \quad \frac{BD}{DC} \quad \text{and replacing} \quad \frac{IE}{BE}$$

by the value given in (3), we obtain

$$\frac{XI}{CX}=\frac{c}{a+b+c}\cdot$$

Adding unity to both sides of this equation gives

$$\frac{XI}{CX}+1=\frac{c}{a+b+c}+1, \quad \text{or} \quad \frac{XI+CX}{CX}=\frac{a+b+2c}{a+b+c}\cdot$$

Inverting both members of this latter equation and we have the first required result

$$\frac{CX}{CI}=\frac{a+b+c}{a+b+2c}\cdot$$

Allow $DE$ extended to meet $AB$ at $G$. The theorem of Menelaus applied to triangle $ABC$ with transversal $EDG$ yields

$$\frac{AG}{GB}\cdot\frac{BD}{DC}\cdot\frac{CE}{EA}=-1 \quad \text{or} \quad \frac{AG}{BG}=\frac{CD}{DB}\cdot\frac{AE}{EC}\cdot$$

Replace

$$\frac{CD}{DB} \text{ by } \frac{b}{c} \text{ and } \frac{AE}{EC} \text{ by } \frac{c}{a} \text{ and we have } \frac{AG}{BG} = \frac{b}{a}.$$

This may be written as

$$\frac{c+BG}{BG} = \frac{b}{a},$$

from which

$$BG = \frac{ac}{b-a}. \tag{4}$$

Now $FG = FB + BG$ and by using (2) and (4) we obtain

$$FG = \frac{2abc}{(a+b)(b-a)}. \tag{5}$$

Using (5) and (4) we have

$$\frac{FG}{BG} = \frac{2b}{a+b}. \tag{6}$$

Let us finally consider triangle $BCF$ with transversal $GDX$. The theorem of Menelaus gives

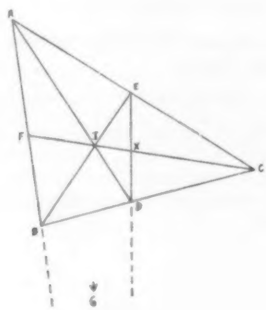
$$\frac{FG}{GB} \cdot \frac{BD}{DC} \cdot \frac{CX}{XF} = -1 \quad \text{or} \quad \frac{CX}{XF} = \frac{CD}{DB} \cdot \frac{BG}{FG}.$$

Substituting

$$\frac{b}{c} \text{ for } \frac{CD}{DB} \text{ and replacing } \frac{BG}{FG}$$

by the value given in (6), gives the second required result

$$\frac{CX}{XF} = \frac{a+b}{2c}.$$




**2715.** *Proposed by Leo Moser, University of Alberta.*

Five lines of general position in a plane meet in 10 points. Show that it is not possible to enter the numbers $1, 2, \cdots, 10$ on these points in such a way that the sum of the numbers along every line is the same.

*Solution by W. R. Talbot, Jefferson City, Mo.*

Let the letters $a$ to $j$ denote the numbers 1 to 10 in some order. Because each number lies on two lines, it is counted twice in summing all five lines. This total is 110 so that the sum on each line must be 22. There is no loss of generality in taking $a=10$. Let the points on the five lines be

| | | |
|---|---|---|
| L1: $10, b, c, d$ | L3: $e, b, h, j$ | L5: $h, c, i, g$. |
| L2: $10, e, f, g$ | L4: $j, d, i, f$ | |

With $b+c+d=e+f+g=12$, we may have only the following pairs of groups:

| | | | | | |
|---|---|---|---|---|---|
| (A) | $1+2+9=3+4+5$ | so that | $h, i, j$ | are | $6, 7, 8$ |
| (B) | $1+3+8=2+4+6$ | so that | $h, i, j$ | are | $5, 7, 9$ |
| (C) | $1+5+6=2+3+7$ | so that | $h, i, j$ | are | $4, 8, 9$. |

There is no second group for $1+4+7=12$. We consider (A), (B), (C), beginning with L3 for which $(b+e)+(h+j)=22$.

I. In (A) we may have $h+j=13$, 14, or 15 so that $b+e=9$, 8, or 7. respectively.

The only possibility is $h+j=15$ with $b+e=2+5=7$. Then $i=6$. In $L4$, $i+j=13$ or 14 and $d+f=9$ or 8 where $d$ is limited to 1 or 9 and $f$ to 3 or 4. The arrangement is impossible for case (A).

II. In (B) we have $h+j=12$, 14, or 16 so that $b+e=10$, 8, or 6 respectively.

The only possibility is $h+j=12$ and $b+e=8+2=10$. Then $i=9$. In L4, $i+j=14$ or 16 so that $d+f=8$ or 6 where $d$ is limited to 1 or 3 and $f$ to 4 or 6. The arrangement is impossible for case (B).

III. In (C) we have $h+j=12$, 13 or 17 so that $b+e=10$, 9, or 5 respectively.

The only possibility is $h+j=13$ and $b+e=6+3=9$. Then $i=8$. In L4, $i+j=12$ or 17 so that $d+f$ must be 10 or 5 where $d$ is limited to 1 or 5 and $f$ to 2 or 7. The arrangement is impossible for case (C).

Interchanging the $b$, $c$, $d$ groups with the $e$, $f$, $g$ groups in (A) to (C) naturally leads to the same type of result and we conclude that the numbers 1 to 10 cannot be distributed to make the sums equal on the five lines.

A solution was also submitted by C. W. Trigg, Los Angeles, Calif.

**2717.** *Taken from Ladies' Diary, 1810.*

What is the smallest square number which, when squared, results in the largest possible succession of equal digits?

*Solution by C. W. Trigg, Los Angeles, Calif.*

Clearly, a one followed by $2n$ zeros is the smallest square number which when squared results in $4n$ equal digits. E.g., $(100)^2=10000$, $(10000)^2=100000000$, etc. If this be considered the trivial case, by searching a table of squares we find the following values of $N^2$ which are the smallest values for which $N^4$ contains $X$ consecutive equal digits:

| $X$ | $N^2$ | $N^4$ |
|---|---|---|
| 1 | 1 | 1 |
| 2 | 256 | 65536 |
| 3 | 4096 | 16777216 |
| 4 | 13924 | 193877776 |
| 5 | No values for $N<1000$. | |

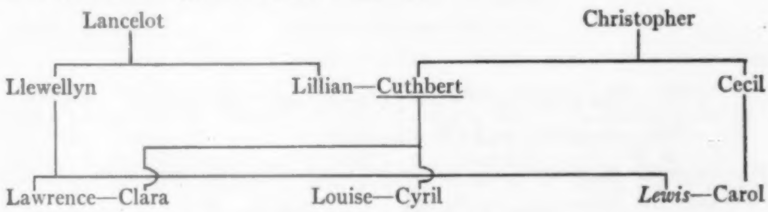
A solution was also submitted by W. R. Talbot, Jefferson City, Mo.

**2718.** *Adapted from Charles L. Dodgson.*

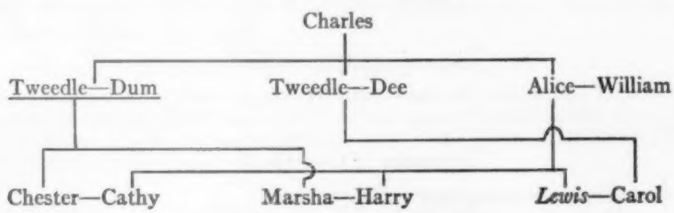
A mathematician gives a small stag party. He invites his father's brother-in-law, his brother's father-in-law, his father-in-law's brother, and his brother-in-law's father. Find the number of guests.

*Solution by C. W. Trigg, Los Angeles, Calif.*

Clearly four guests could have been invited, so we assume that the implied question is "what is the smallest number of guests possible?" Upon reference to the family tree of the descendents of Lancelot and Christopher it is evident that mathematician Lewis could have had only Cuthbert at his party, if a couple of first-cousin-marriages had been countenanced.




If tighter inbreeding is permitted as in the case of the progeny of Charles, then uncle Tweedle-Dum could have been Lewis' sole guest.




A solution was also submitted by W. R. Talbot, Jefferson City, Mo.

**2719**. *Proposed by Cecil B. Read, Wichita, Kans.*

Given that $a$ is quite small as compared to $b$ and $c$, find an approximation to the numerically smaller root of the quadratic equation $ax^2+bx+c=0$.

*Solution, by the proposer*

As $a$ approaches zero, the given equation is approximately $bx+c=0$, hence a first approximation is $-c/b$. For a better approximation we note that since $a$ is small, the numerically smaller root will be given if we choose the positive sign for the radical in the usual quadratic formula, i.e., the desired root is

$$(-b+\sqrt{b^2-4ac})/2a.$$

$$(b^2-4ac)^{1/2}=b\left(1-\frac{4ac}{b^2}\right)^{1/2}=b\left(1-\frac{1}{2}\,\frac{4ac}{b^2}-\frac{1}{8}\,\frac{16a^2c^2}{b^4}-\cdots\right)$$

$$=b-\frac{2ac}{b}-\frac{2a^2c^2}{b^3}-\cdots$$

Hence a more accurate approximation is given by

$$x=-\frac{c}{b}-\frac{ac^2}{b^3}-\cdots$$

Solutions were also submitted by C. W. Trigg, Los Angeles, Calif.; and Dale Woods, Kirksville, Mo.

**2720.** *Proposed by Walter R. Talbot, Jefferson, City, Mo.*

Evaluate the real positive function:

$$\sin \arctan \sec \operatorname{arccsc} \cot \arccos \csc \operatorname{arccot} \cos \arcsin \tan \operatorname{arcsec} y.$$

*Solution by C. W. Trigg, Los Angeles, Calif.*

If there is an

$$\operatorname{arcsec} y, \qquad \text{then } |y| \geqq 1.$$

then

$$\tan \operatorname{arcsec} y = \pm\sqrt{y^2-1},$$

so

$$\cos \arcsin(\pm\sqrt{y^2-1}) = \pm\sqrt{2-y^2}, \quad \text{and} \quad |y| \leqq \sqrt{2}.$$

$$\csc \operatorname{arccot},(\pm\sqrt{2-y^2}) = \pm\sqrt{3-y^2}.$$

But the cosine of an angle may not exceed one in absolute value, so in

$$\arccos(\pm\sqrt{3-y^2}), \qquad |y| \text{ must be } \sqrt{2},$$

and

$$\arccos(\pm 1) = 0 \quad \text{or} \quad \pi.$$

Then

$$\cot \arccos(\pm 1) \text{ is undefined,}$$

$$\sec \operatorname{arccsc} (\text{undefined}) \text{ is } \pm 1.$$

Finally

$$\sin \arctan(\pm 1) = \pm(\tfrac{1}{2}\sqrt{2}).$$

A solution was also submitted by the proposer.

**2721.** *Proposed by Brother Felix John, Philadelphia, Pa.*

The sides of a right triangle are $a$, $b$, and $c$, and the areas of the triangle is 84 square units. The sides of a second triangle (not right), are $d$, $e$, and $f$, and these sides form an arithmetic progression with a common difference of 1. if $d$ is 1 less than twice $a$, and $e$ and $f$ are each 10 less than $b$ and $c$ respectively, find $a$, $b$, $c$, $d$, and $e$.

*Solution by Hermann Boeckmann, Bloomington, Ill.*

I. Given information:

| | |
|---|---|
| $168=ab$ | $d=d$ |
| $d=2a-1$ | $e=d+1$ |
| $e=b-10$ | $f=d+2$ |
| $f=c-10$ | |

II. Solving for $a$ in terms of $d$:

$$2a=d+1 \quad \text{or} \quad a=\tfrac{1}{2}(d+1)$$

III. Solving for $b$ in terms of $d$:

$$d+1=b-10 \quad \text{or} \quad b=d+11$$

IV. Solving for $c$ in terms of $d$:

$$d+2=c-10 \quad \text{or} \quad c=d+12$$

V. Therefore,

$$168=\tfrac{1}{2}(d+1)(d+11) \quad \text{or} \quad d^2+12d-325=0$$

VI. Since

$$a^2+b^2=c^2$$

and make the proper substitutions from II, III, and IV

$$d^2-6d-91=0$$

VII. Therefore,

$$d^2+12d-325=d^2-6d-91 \quad \text{or} \quad d=13$$

VIII. Hence

$$a=7, \quad b=24, \quad c=25, \quad d=13, \quad e=14, \quad \text{and} \quad f=15$$

Solutions were also offered by Louis S. Cohen, Los Angeles, Calif.; Brother A. Faber, Lafayette, La.; Herta T. Freitag, Roanoke, Va.; Mary Harris, Jefferson City, Mo.; Diane Kosowski, Chicago, Ill.; Richard Lamb, Naperville, Ill.; Herbert Leifer, Pittsburg, Pa.; Sister M. Lucine, Hamburg, N. Y.; W. R. Talbot, Jefferson City, Mo.; Eugene Togan, Detroit, Mich.; C. W. Trigg, Los Angeles, Calif.; Kenneth Wilkins, Detroit, Mich.; Dale Woods, Kirksville, Mo.; and the proposer.

**2722.** *Proposed by J. W. Lindsey, Amarillo, Texas.*

Find the equation of the circle tangent to the lines $y-3x=20$ and $x=3y=10$ and passing through the origin.

*Solution by Herta T. Freitag, Roanoke, Va.*

The required circles have equation $x^2+y^2-2hx-2ky=0$, where $M(h, k)$ symbolizes their centers, Since $M$ must lie on the angle bisector of the given lines, $2h+k+5=0$, (1). The distance of $M$ from each of the given lines must be equal to the radius $r$, and — since

$$r^2=h^2+k^2-\frac{3h-k+20}{\sqrt{10}}=\sqrt{h^2+k^2} \tag{2}$$

Thus, $M_1(5, -15)$ and $M_2(-3, 1)$. The required equations are

$$x^2+y^2-10(x-3y)=0 \quad \text{and} \quad x^2+y^2+2(3x-y)=0$$

Solutions were also submitted by R. A. Baumgartner, Freeport, Ill.; Louis Cohen, Los Angeles, Calif.; Herman Boeckmann, Bloomington, Ill.; J. Byers King, Denton, M.D.; C. N. Mills, Sioux Falls, S. D.; Eugene Togan, Detroit, Mich.; W. R. Talbot, Jefferson City, Mo.; C. W. Trigg, Los Angeles, Calif.; Dale Woods, Kirksville, Mo.; and the proposer.

**2723.** *Taken from Mathematical Pie.*

Three fierce dogs, Alpha, Beta, and Gamma, stand at the vertices of a large equilateral triangle. At a given signal, Alpha chases Beta, Beta chases Gamma and Gamma chases Alpha, all with the same speed. Sketch the paths run by each of the dogs.

*Solution by Kenneth Wilkins, Detroit, Mich.*

Since all the dogs run at the same rate, each dog approaches the other at the same rate. Therefore, since they started an equal distance apart, the dogs will always remain an equal distance apart. This means that at any given moment the three dogs will always form the corners of an equilateral triangle, which shrinks and rotates as the dogs move closer together. The actual path taken by each dog is a logarithmic spiral. Since each dog travels a similar spiral, they must all meet in the center of the triangle.

A solution was also submitted by Herman Boeckmann, Bloomington, Ill.

**2724.** *Proposed by Walter R. Talbot, Jefferson City, Mo.*

Clocks $A$ and $B$ were started at the correct time but $A$ loses 5 seconds every true hour and reads 10:49, and $B$ gains 7 seconds every true hour and reads 2:01. What is the correct time and how long has it been since the clocks were in agreement? How long will it be before the clocks next read the same and what time will they show?

*Solution by Mary Harris, Jefferson City, Mo.*

Each true hour there is a change by 12 seconds in the readings of the clocks. The clocks now differ by $3\frac{1}{5}$ hours which means they have been running for 960 hours. $A$ is, therefore, 1 hour and 20 minutes behind the correct time and $B$ is 1 hour and 52 minutes ahead of the correct time. The correct time is 12:09. It has been 960 hours since the clocks were in agreement. When the clocks are 12 hours apart, 3600 hours from starting time, they will again read the same. This will be 2640 true hours from their present readings. In 2640 hours (exactly 110 days) $A$ will read 10:49 less 3 hours and 40 minutes and $B$ will read 2:01 plus 5 hours and 8 minutes, or both will read 7:09.

Solutions were also submitted by Herman Boeckmann, Bloomington, Ill.; Enoch J. Haga. Vacaville, Calif.; J. Byers King, Denton, Md.; C. W. Trigg, Los Angeles, Calif.; and the proposer.

## HIGH SCHOOL HONOR ROLL

**The Editor will be very happy to make special mention of high school classes, clubs, or individual students who offer solutions to problems submitted to this department. Teachers are used to report to the Editor such solutions.**

**Editor's Note: For a time each high school contributor will receive a copy of the magazine in which the student's name appears.**

**For this issue the Honor Roll appears below.**

**2713, 2721.** *Lee H. Mitchell, Ann Arbor, Mich.*

**2715.** *Jack Halliburton, Los Angeles, Calif.*

**2716, 2722, 2723.** *Barabara Jannusch, Visitation High School Chicago, Ill.*

**2721.** *Elizabeth Dusch, Beaumont High School, St. Louis, Mo.*

**2720, 2724.** *Donald Muench, McQuaid High School, Rochester, N. Y.*

**2721.** *Isa Goldman, West Coast Talmudical Seminary, North Hollywood, Calif.*

**2724.** *Craig Nielsen, Oradell, N. J.*

## SOLUTIONS TO STUDENT PROBLEMS

**S-1.** No Solution has been offered.

**S-2.** *Proposed by J. W. Lindsey, Amarillo, Texas.*

The base of a triangle is given in magnitude and position and the difference of the squares on the other two sides also is given. Find the locus of the vertex.

*Solution by Patricia Murphy, Visitation High School, Chicago.*

Let the coordinate axes be chosen with the $x$-axis along the segment $P_1P_2$. Then the $y$-coordinates of $P_1$ and $P_2$ will be zero and the $x$-coordinates will be 0 and $a$. I chose the numbers $(a, 0)$ as the coordinates of $P_1$, the coordinates of $P_2$ will be $(0, 0)$. Let $P(x, y)$ be an arbitrary point of the locus.

$P$ must satisfy the condition:

$$(P_1P)^2-(P_2P)^2=k$$

$$(P_2P)^2-(P_1P)^2=k$$

By the formula for distance between two points we have:

$$P_1P=\sqrt{(x-a)^2+y^2}$$

$$P_2P=\sqrt{(x-0)^2+y^2}$$

Substituting the values of the first equation we have:

$$(x-a)^2+y^2-(x-0)^2-y^2=k$$

$$x^2-2ax+a^2+y^2-x^2-y^2=k$$

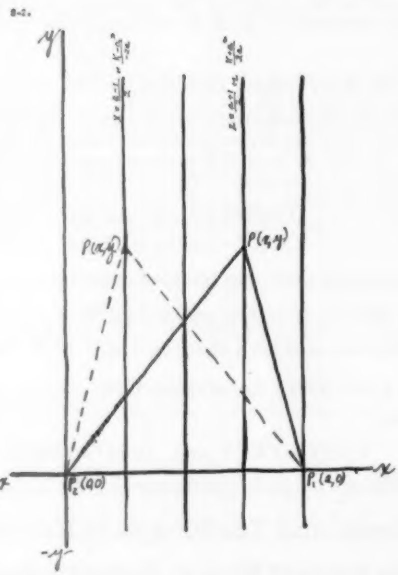
$$-2ax+a^2=k$$

$$-2ax=k-a^2$$

$$x=\frac{k-a^2}{-2a}$$

Since $k=a$ $$x=\frac{a-a^2}{-2a}$$

(1) $$x=\frac{a-1}{2}$$




Substituting the values of the second equation we have:

$$(x-0)^2+y^2-(x-a)^2-y^2=k$$

$$x^2+y^2-x^2+2ax-a^2-y^2=k$$

$$2ax-a^2=k$$

$$2ax=k+a^2$$

$$x=\frac{k+a^2}{2a}$$

Since $k=a$ $$x=\frac{a+a^2}{2a}$$

(2) $$x=\frac{a+1}{2}$$

The locus of vertex $P$ is one of the parallel lines given by equations (1) & (2)

## PROBLEMS FOR SOLUTION

**2743.** *Proposed by C. W. Trigg, Los Angeles, Calif.*

Find the smallest pair of consecutive integers, each of which can be expressed as the sum of two cubes.

**2744.** *Proposed by Lowell Van Tassel, San Diego, Calif.*

Here is a probability problem that was suggested by a recent promotional stunt for *Time* magazine. It seems that every month, for the next 12 months, a small postcard-sized picture of a former MAN OF THE YEAR will be enclosed with the magazine on a random basis. (This will happen only once a month even though *Time* is published weekly). Problem: What are the odds on collecting a complete 12 man set in the next year?

**2745.** *Proposed by John Satterly, University of Toronto*

Prove that the center of gravity of three masses proportional to sin $2A$, sin $2B$, sin $2C$ placed at the vertices $A$, $B$, $C$ respectively at a triangle $ABC$ is at the circumcenter.

**2746.** *Proposed by D. Moody Baily, Princeton, W. Va.*

$P$ is any point in the plane of triangle $ABC$ through which cevians are drawn from $A$, $B$, and $C$ meeting the opposite sides at points $D$, $E$, and $F$, respectively. $M$ is the midpoint of side $BC$ and $MP$ is constructed meeting $CA$ at $N$ and $AB$ at 0. Show that

a) $$AO/OB=AF/FB-AE/EC$$

b) $$AN/NC=AE/EC-AF/FB$$

where all segments are considered as directed quantities.

**2747.** *Proposed by Thomas R. Curry, Oyster Bay, N. Y.*

There exists a number $abcd$ such that $(ab+cd)^2=abcd$. Find the number.

**2648.** *Proposed by C. W. Trigg, Los Angeles, Calif.*

In the scale of six,

$$TIRES=(TEE)^2 \quad \text{and} \quad IREST=(EET)^2.$$

Find the digits which are uniquely represented by the letters.

## Books and Teaching Aids Received

EXPLORING SCIENCE SERIES, by Walter A. Thurber *Professor of Science, Cortland State Teachers College, Cortland, New York.* All cloth. All 16×21.5 cm. 1960. Allyn and Bacon, Inc., Tremont Street, Boston, Massachusetts
- Book Four, 264 pages+144 (teachers' edition).
- Book Five, 360 pages+176 (teachers' edition).
- Book Six, 360 pages+160 (teachers' edition).

MATHEMATICS FOR ENGINEERS (2 volumes), by W. N. Rose, *Late Head of the Mathematics Department at the Borough Polytechnique.* Both cloth. Both 21.5 ×13.5 cm. 1960. John F. Rider Publisher, Inc., 116 West 14th Street, New York 11, N. Y.
- Volume I, xiv+527 pages. Price $6.60.
- Volume II, xii+403 pages. Price $6.60.

REVIEW OF MATHEMATICS, by F. L. Minnear and Ruby M. Grimes. Paper. 11×18 cm. 198 pages. 1960. W. H. Freeman and Company, 660 Market Street, San Francisco 4, California. Price $1.65.

BIOLOGY, by Elsbeth Kroeber, Walter H. Wolff, and Richard L. Weaver. Cloth. 23.5×16 cm. 646 pages. 1960. D. C. Heath and Company, 285 Columbus Avenue, Boston 16, Massachusetts. Price $5.32.

MASS LENGTH AND TIME, by Norman Feather FRS. Cloth. 21.5×14 cm. Pages ix+358. 1960. Quadrangle Books, Inc., 119 West Lake Street, Chicago 1, Illinois. Price $5.00.

THEORY AND SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS, by Donald Greenspan, *Purdue University*. Cloth. 23.5×16 cm. 147 pages. 1960. The MacmillanCompany, 60 Fifth Avenue, New York 11, N.Y. Price $5.50.

THE EDGE OF OBJECTIVITY, by Charles Coulston Gillispie. Cloth 21.5×14 cm. 562 pages. 1960. Princeton University Press, Princeton, New Jersey. Price $7.50.

THE RETARDED CHILD GOES TO SCHOOL, by Harold M. Williams, *Specialist, Education for Exceptional Children and Youth*. Paper. 23×15 cm. 24 pages. U. S. Department of Health, Education, and Welfare, Office of Education, Washington, D. C. Price $.15.

RECHT CHEM-FORMULATOR, by Christian F. Recht. Cardboard. Chemistry Wheels. Front-properties of elements, Back-inorganic radicals and formulas. 8×8 in. 1959. Recht Chem-Formulator, Company, Box 225, Boulder, Colorado Price $.50 postpaid.

PAPERBOUND BOOKS IN THE HISTORY AND PHILOSOPHY OF SCIENCE, compiled by L. E. Klopfer. Paper. 28×21.5 cm. 16 pages. Harvard University, Graduate School of Education, 72 Batchelder House, Kirkland Street, Cambridge 38, Massachusetts. Price $.10.

HUMAN DEVELOPMENT, by Phyllis C. Matin, *University of Pittsburgh*, and Elizabeth Lee Vincent, *Chatham College*. Cloth. 15×23 cm. v 541 pages. 1960. The Ronald Press Company, 15 East 26th Street, New York 10, N. Y. Price $8.00.

SUGGESTED SCIENCE BOOKS FOR THE PUPIL AND TEACHER. Paper. 32 pages. 23×15 cm. College of Education, State University of Iowa, Iowa City, Iowa. 1960.

U. S. GOVERNMENT GRANTS UNDER THE FULBRIGHT AND SMITH-MUNDT ACTS. Program Announcements 1961–1962. Paper. 64 pages. 21.5×14 cm. 1960. Conference Board of Associated Research Councils, Committee on International Exchange of Persons, 2101 Constitution Avenue, Washington 25, D.C.

WHAT IS IT SERIES? All cloth. All 48 pages. All 18.5×19.5 cm. All $1.60. 1960. Benefic Press, 1900 N. Narragansett, Chicago 39, Illinois.
- WHAT IS A BIRD, by Gene Darby.
- WHAT IS A PLANT, by Gene Darby.
- WHAT IS A SEASON, by Gene Darby.
- WHAT IS A TURTLE, by Gene Darby.
- WHAT IS A FISH, by Gene Darby.
- WHAT IS A CHICKEN, by Gene Darby.
- WHAT IS A COW, by Gene Darby.
- WHAT IS A ROCK, by B. John Syrocki.
- WHAT IS A ROCKET, by Theodore W. Munch, Ed.D.
- WHAT IS A MAGNET, by Gabriel H. Reuben and Gloria Archer.
- WHAT IS A MACHINE, by B. John Syrocki.

WHAT IS LIGHT, by Theodore W. Munch, Ed.D.
WHAT IS A SOLAR SYSTEM, by Theodore W. Munch, Ed.D.
WHAT IS A FROG, by Gene Darby.
WHAT IS A TREE, by Gene Darby.
WHAT IS A BUTTERFLY, by Gene Darby.

SCIENCE AND CONSERVATION SERIES. Both paper. 1960. Benefic Press, 1900 N. Narragansett, Chicago 39, Illinois.
Teacher's Manual for LET'S FIND OUT, LET'S LOOK AROUND, LET'S SEE WHY, by Samuel A. Thorn and Irene Harbeck. 64 pages. 15.5×21 cm.
Teacher's Manual for LET'S LEARN HOW, LET'S KNOW NOW, LET'S DISCOVER MORE, by Samuel A. Thorn and Carl D. Duncan. 144 pages. 15.5×21 cm.

SCIENCE AND CONSERVATION SERIES. All cloth. All 15×21 cm. 1960. Benefic Press, 1900 N. Narragansett, Chicago 39, Illinois.
LET'S GO, by Samuel A. Thorn and Jeanne Brouillette. 48 pages. Price $1.52.
LET'S TRY, by Samuel A. Thorn and Jeanne Brouillette. 72 pages. Price $1.52.
LET'S FIND OUT, by Sam Thorn and Irene Harbeck. 128 pages. Price $1.72.
LET'S LOOK AROUND, by Samuel A. Thorn and Irene Harbeck. 159 pages. Price $2.00.
LET'S SEE WHY, by Samuel A. Thorn and Carl D. Duncan. 192 pages. Price $2.20.
LET'S LEARN WHY, by Samuel A. Thorn and Carl D. Duncan. 255 pages. Price $2.52.
LET'S KNOW NOW, by Samuel A. Thorn and Carl D. Duncan. 287 pages. Price $2.60.
LET'S DISCOVER MORE, by Samuel A. Thorn and Carl D. Duncan. 317 pages. Price $2.80.

BASIC ULTRASONICS, by Cyrus Glickstein. Paper. 15.5×23 cm. 144 pages. 1960. John F. Rider Publisher, Inc., 116 West 14 Street, New York, N. Y. Price $3.50.

The Story of BUTTERFLIES and other INSECTS, by Peter Farb. Cloth. 126 pages. 18.5×25 cm. 1960. Harvey House, Publishers, Irvington-On-Hudson, New York. Price $2.95.

The Story of PLANETS, SPACE, and STARS, by Gaylord Johnson. Cloth. 153 pages. 18.5×25 cm. 1960. Harvey House, Publishers, Irvington-On-Hudson, N. Y. Price $2.95.

MODERN COLLEGE ALGEBRA, by Julian D. Mancill, *University of Alabama*, and Mario O. Gonzalez, *University of Havana and University of Alabama*. Cloth. 15×22.5 cm. xi+386 pages. 1960. Allyn and Bacon, Inc., 150 Tremont St., Boston 11, Mass. Price $6.25.

ELEMENTARY MODERN PHYSICS, by Richard T. Weidner and Robert L. Sells. Cloth. 513 pages. 15×23 cm. 1960. Allyn and Bacon, Inc., 150 Tremont St., Boston 11, Mass. Price $8.50.

ATOMS, MOLECULES, AND CHEMICAL CHANGE, by Ernest Grunwald and Russel H. Johnson. 15×23 cm. Cloth. xii+252 pages. 1960. Prentice-Hall, Inc., 70 Fifth Ave., New York 11, New York. Price $6.00.

MATTER, EARTH, AND SKY, by George Gamow. Cloth. 593 pages. 15.5×22.5 cm. 1960. Prentice-Hall, Inc., 70 Fifth Ave., New York 11, New York. Price $6.00.

RADIO CONTROL FOR MODEL BUILDERS, by William Winter. Paper. 228 pages. 21×14 cm. 1960. John F. Rider Publisher, Inc., 116 West 14 Street, New York, N. Y. Price $4.25.

# Book Reviews

College Mathematics for Freshmen, by Paul K. Smith, *Professor of Mathematics, Louisiana Polytechnic Institute*, and Henry F. Schroeder, *Professor of Mathematics, Louisiana Polytechnic Institute*. Cloth. Pages x+314. 15×23 cm. 1959. D. Van Nostrand Company, Inc., 120 Alexander Street, Princeton, New Jersey. Price $4.25.

The authors state this text in general mathematics is designed for students who do not require intensive training in mathematical techniques. They feel that needs of such students will be better served by a course based on this text than by traditional courses. The material includes a review of arithmetic, algebra through simultaneous quadratics, statistical and algebraic graphs. There is extensive treatment of mathematics of finance, two chapters on numerical trigonometry, and some material on solid geometry (largely mensuration, no formal proofs). There is sufficient material for two three semester hour courses, although in a few places the number of problems seems inadequate.

Whether or not the text is satisfactory will depend on the objectives of the course. In the opinion of the reviewer, there is far too much space devoted to mechanical rules, with little attention to giving a student any understanding of the processes. Some treatments seem extremely brief, for example: slightly over one page devoted to the binomial formula; complex numbers treated in a quarter page of text.

In a few places the treatment is better than in the average text. The analysis of the solution of worded problems seems exceptionally detailed and should be helpful. Logarithmic solution of triangles is omitted, which is commendable since few industries now utilize logarithmic methods when calculators are available. (One wonders why some 30 pages of tables of logarithms of trigonometric functions are included.)

At some places the reviewer feels that there are incorrect or confusing statements. It is not clear why (p. 5) in subtracting decimals zeros should be annexed to make both numbers have the same number of decimal places, but this is not considered necessary in addition. Does one raise a number to an exponent or to a power (p. 42)? The authors are not always consistent in restricting a divisor to non-zero numbers (pp. 45, 125, for example). Does one "solve a formula" (p. 49)? It seems odd to speak of the sign *on* a term of a fraction (p. 56). According to the statement on page 88 a system of first degree equations in $x$ and $y$ must have numerical coefficients. There is no mention of the existence of alternative methods for evaluating a third order determinant. The definition of function includes multiple valued functions; some teachers prefer the term "relation," restricting "function" to the single valued case. According to the text (p. 131) it would appear that $\sqrt[6]{x^2}=3$ is *not* an irrational equation. In the illustrative example on pages 134–5 there is no reason given for discarding a negative root—perhaps the reason is obvious. The statement that the mantissa of a logarithm is not changed by multiplying the number by a power of ten is not true if one uses the authors' definition of a mantissa ($\log 0.05=-1.3010$; by p. 149 the mantissa is $-.3010$). Many teachers will object to the statement (p. 184) that in a ratio the units need not be the same. Does a list of six trigonometric functions constitute a *complete* list (p. 194)? The versed sine or haversine have uses, although less frequently encountered.

If the teacher wants a "cook book" type of text, this may fill the bill. The reviewer does not feel it would lead to any great understanding of basic mathematical principles or of the reasons underlying rules of procedure.

Cecil B. Read
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*The Scientific American Book of Mathematical Puzzles and Diversions*, by Martin Gardner. Cloth. Pages xi+178. 21×14 cm. 1959. Simon and Schuster, Inc., Rockefeller Center, 630 Fifth Avenue, New York 20, N. Y. Price $3.50.

This is an excellent book on mathematical recreations. Its particular value seems to the reviewer to be the fact that is aimed at a higher lever of mathematical maturity than many works, which often deal with little more than "number tricks." Some of the material is quite modern, for example the paper folding material in the chapter "Hexaflexagons." Magic squares are discussed as square matrices; ticktacktoe is discussed in two, three and four dimensions; there is a chapter on topological models which goes far beyond the usual brief treatment of a Moebius strip.

By all means this book should be in the high school or college library, to say nothing of the private library of the teacher. Although as stated it is somewhat more advanced than some books on recreations, essentially everything could be followed by an interested high school student.

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